Applications of dynamic games: Bargaining and reputation

Lectures in Game Theory
Fall 2012, Lecture 4

Modeling of

- negotiations between parties with opposing interests:
  Sequential bargaining games

- non-cooperative cooperation between parties that in the short run want to deviate:
  Repeated games

Bargaining

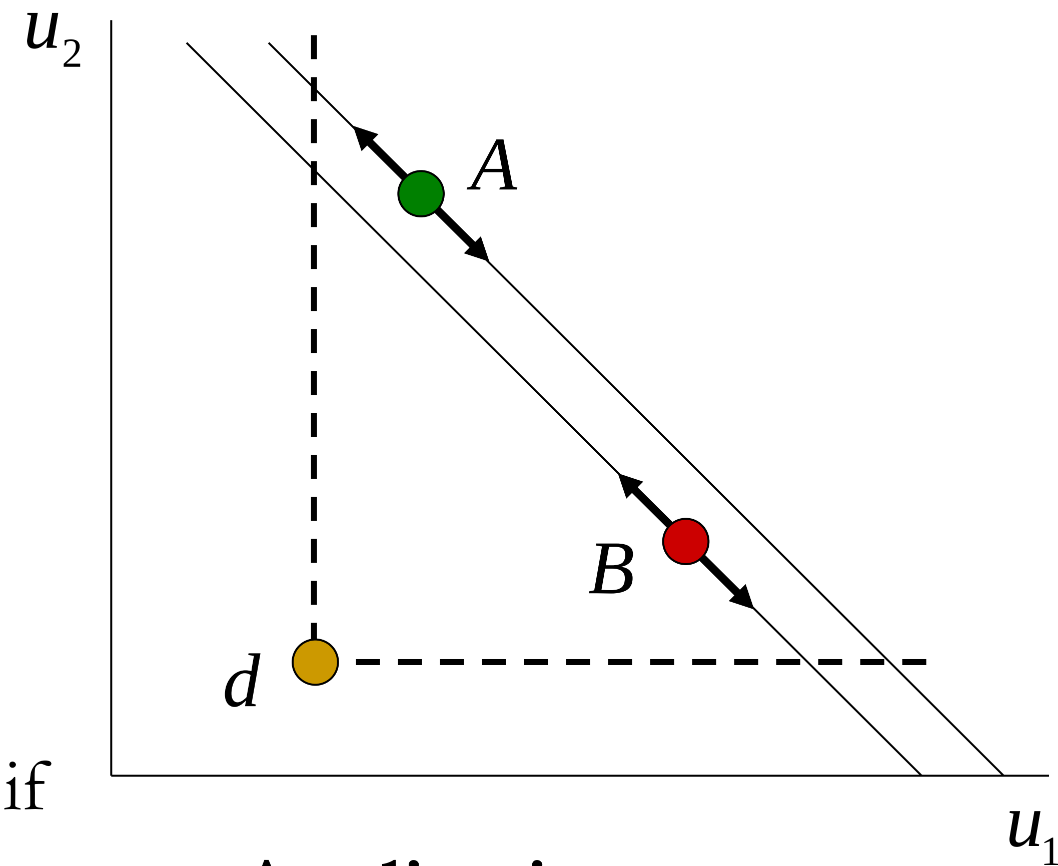
- **Agreement** creates value, but how to divide?

  d: default outcome, disagreement point,

  A, *B*: outcomes

  A: efficient outcome if transfers are possible

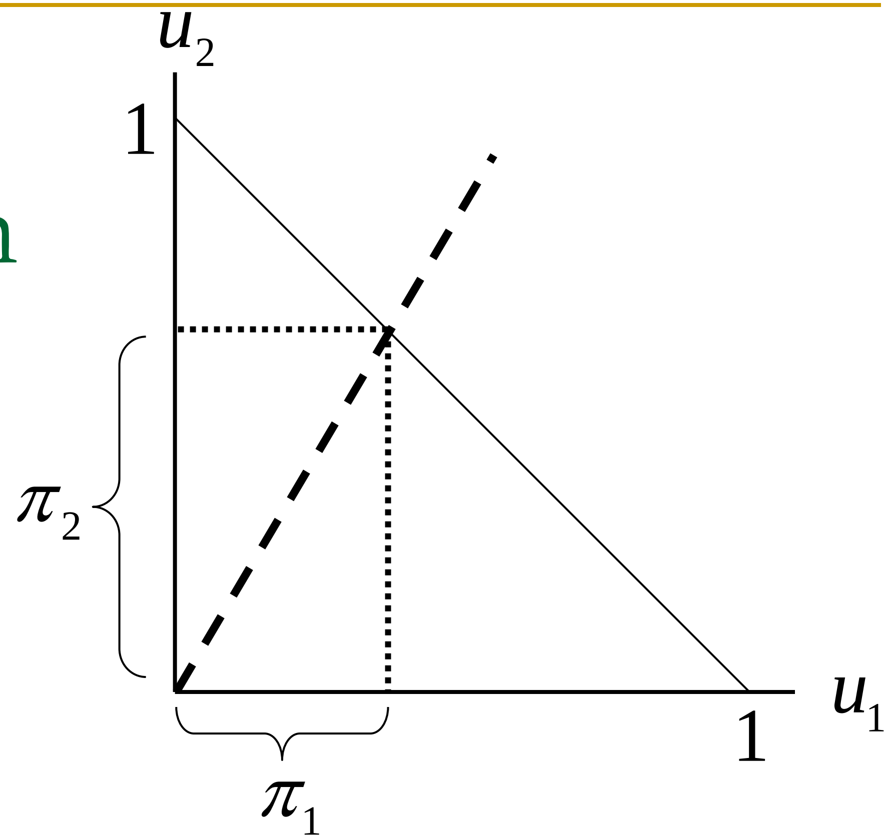
- **Efficient bargaining:** Agree on efficient out-come and divide surplus.



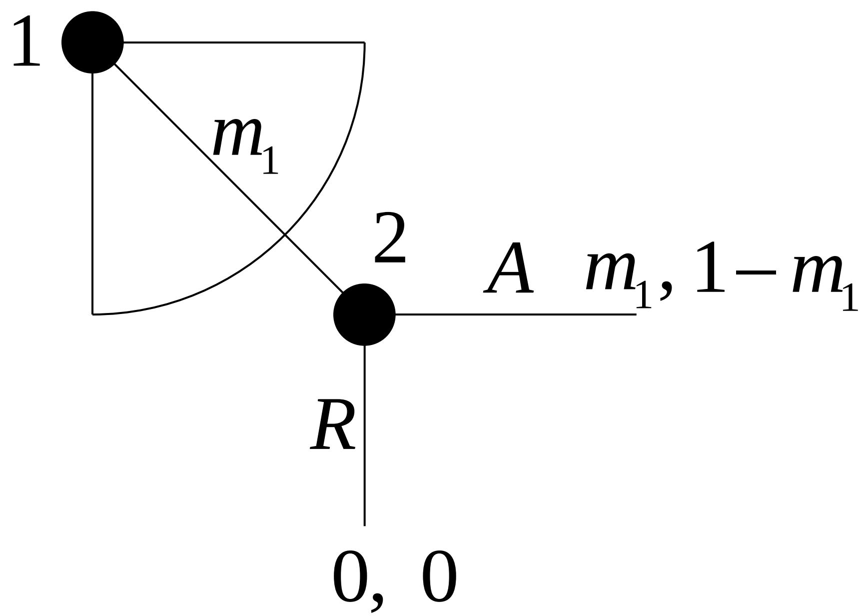
- **Applications:**
  Bilateral monopoly
  Labor market negotiations
  International negotiations

The standard bargaining solution

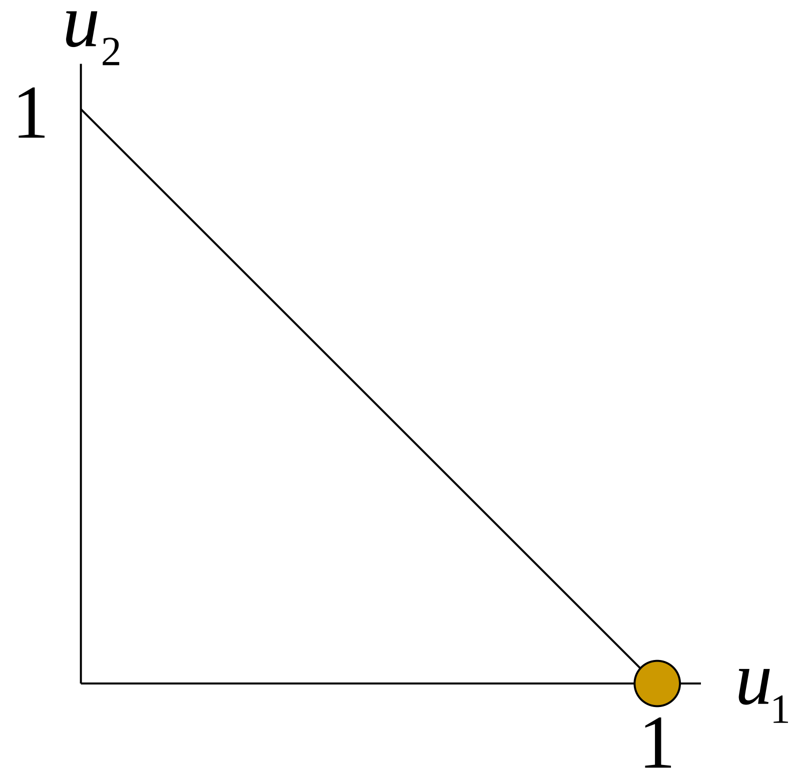
- First: Normalize.
- Then: Divide surplus according to bargaining weights, π_1 and π_2.



- What do the bargaining weights depend on?



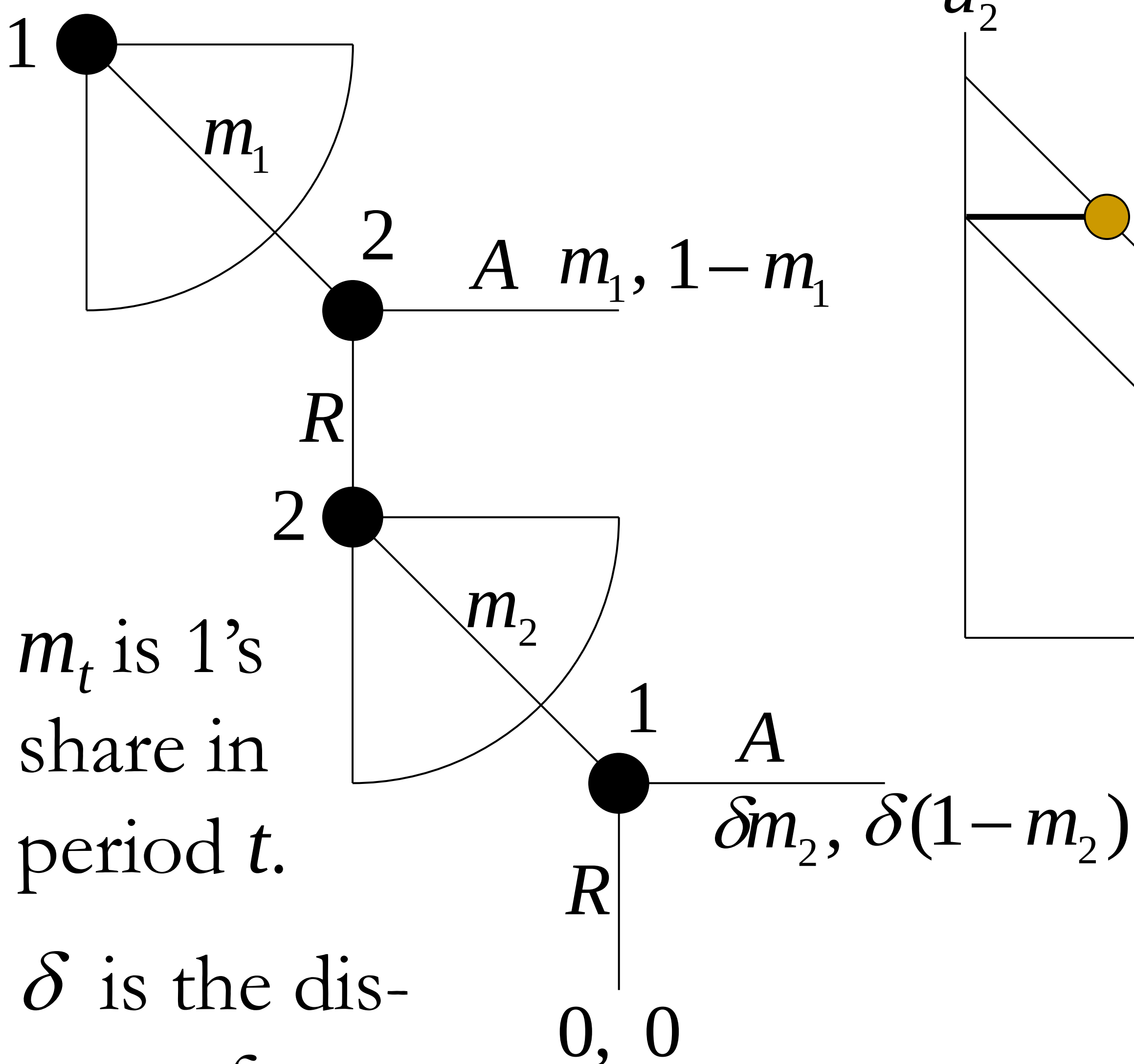
m_1 is 1's share.

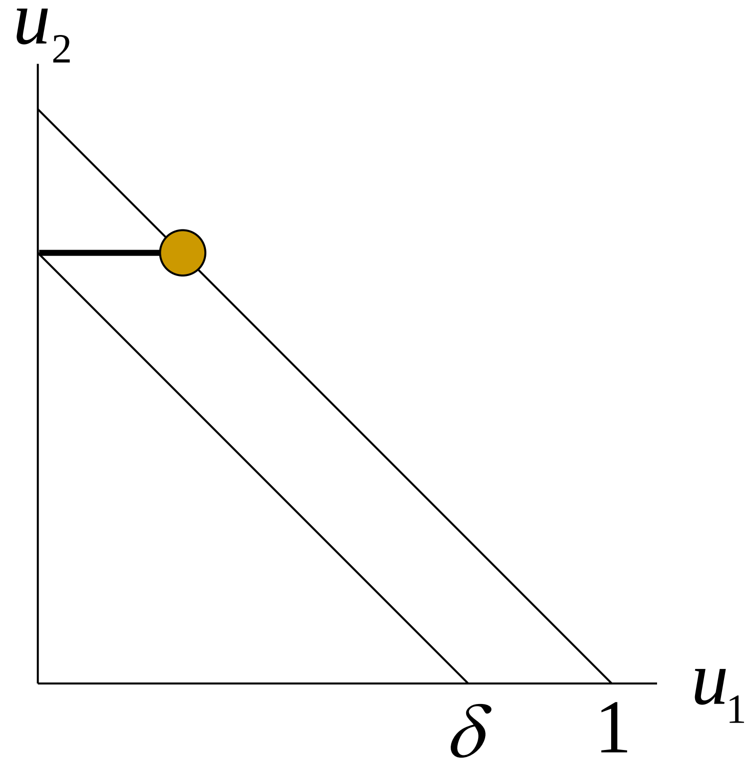


Ultimatum bargaining

- Bargaining weight = 1 for the proposer

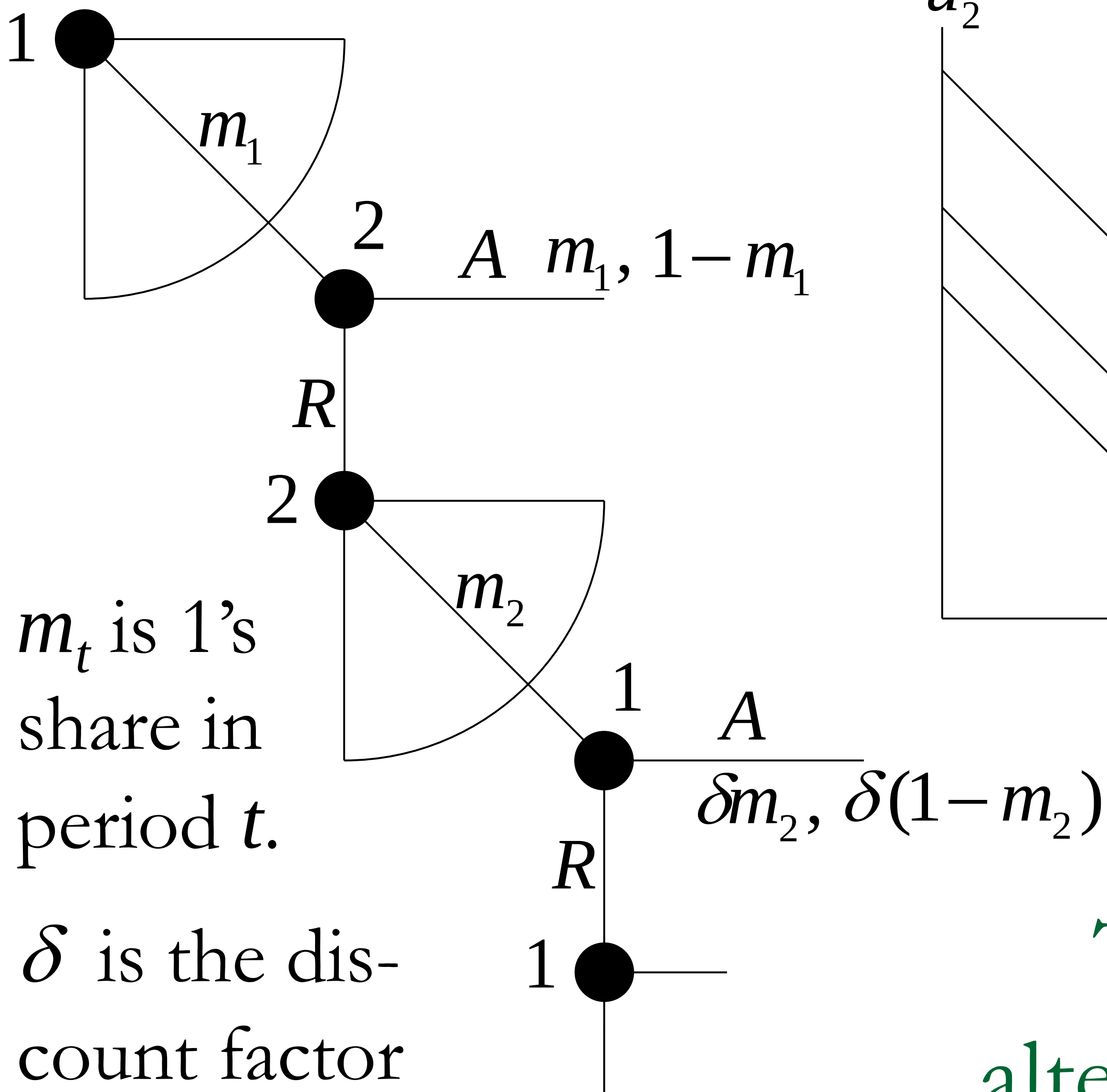
Two-period alternating offer





m_t is 1's share in period t.

δ is the discount factor



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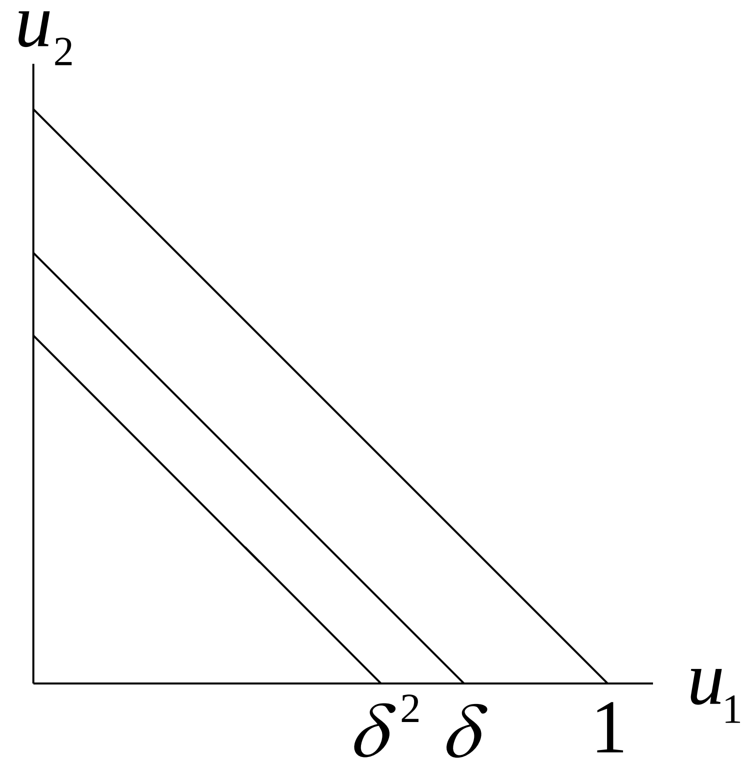
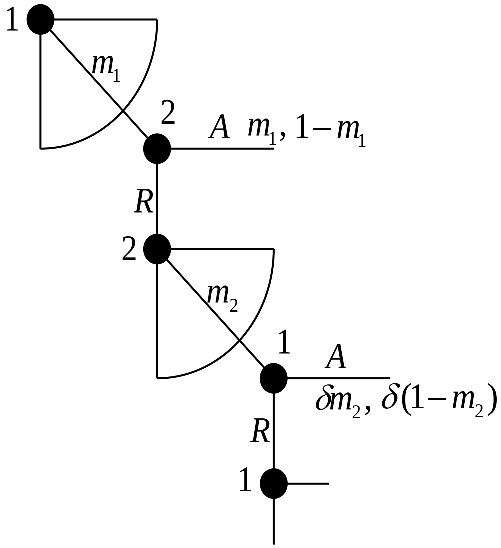


Three-period alternating offer



Backward induction:

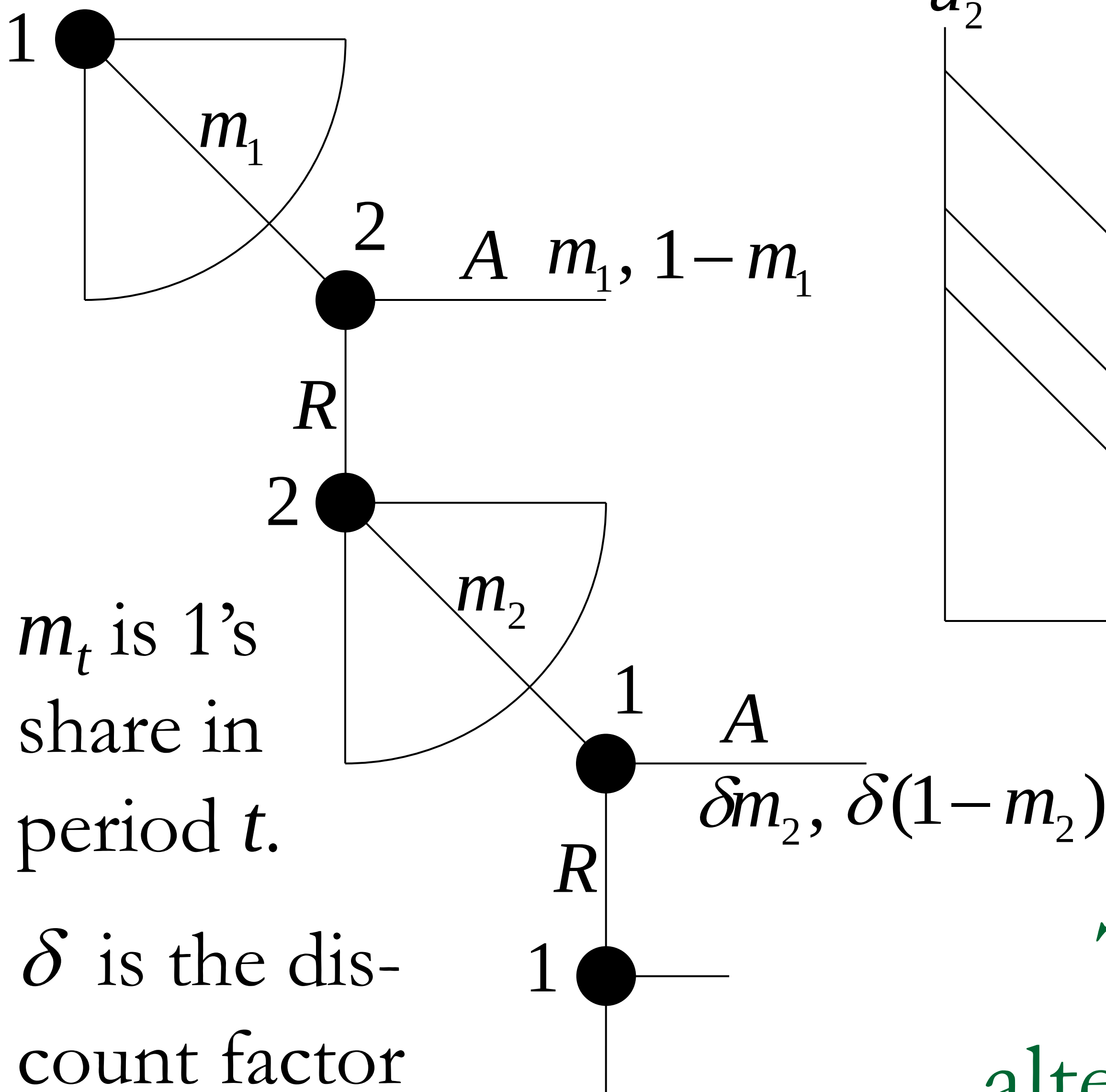
Period 3: 1 offers $m_3 = 1$, 2 will accept. Payoff is δ^2 , 0

Period 2: 1 indiff between accepting and rejecting if $m_2\delta = \delta^2$. Hence 2 will offer $m_2 = \delta$, *1* will accept. Payoff is δ^2 and $(1-m_2)\delta = (1-\delta)\delta$

Period 1: 2 indiff between accepting and rejecting if $1-m_1 = (1-\delta)\delta$. Hence, if 1 will offer $m_1 = 1-\delta(1-\delta)$ then 2 will accept. Will 1 want to offer this? Yes since $m_1 > \delta^2$

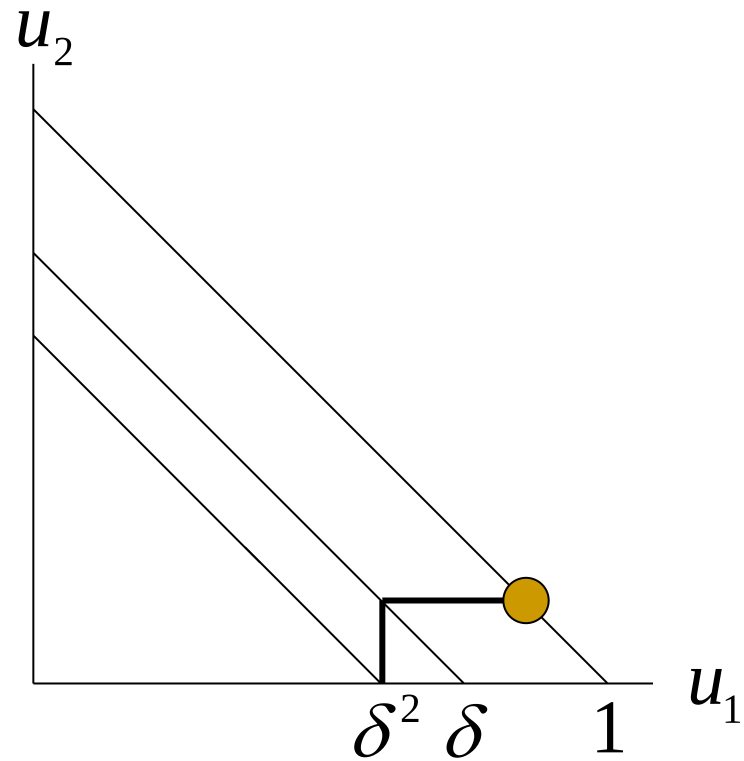
SPNE: $s_1 = [m_1 = 1-\delta(1-\delta)$, A if $\delta \le m_2$, $m_3 = 1]$
$s_2 = [A$ if $m_1 \le 1-\delta(1-\delta)$, $m_2 = \delta$, A if $m_3 \le 0]$

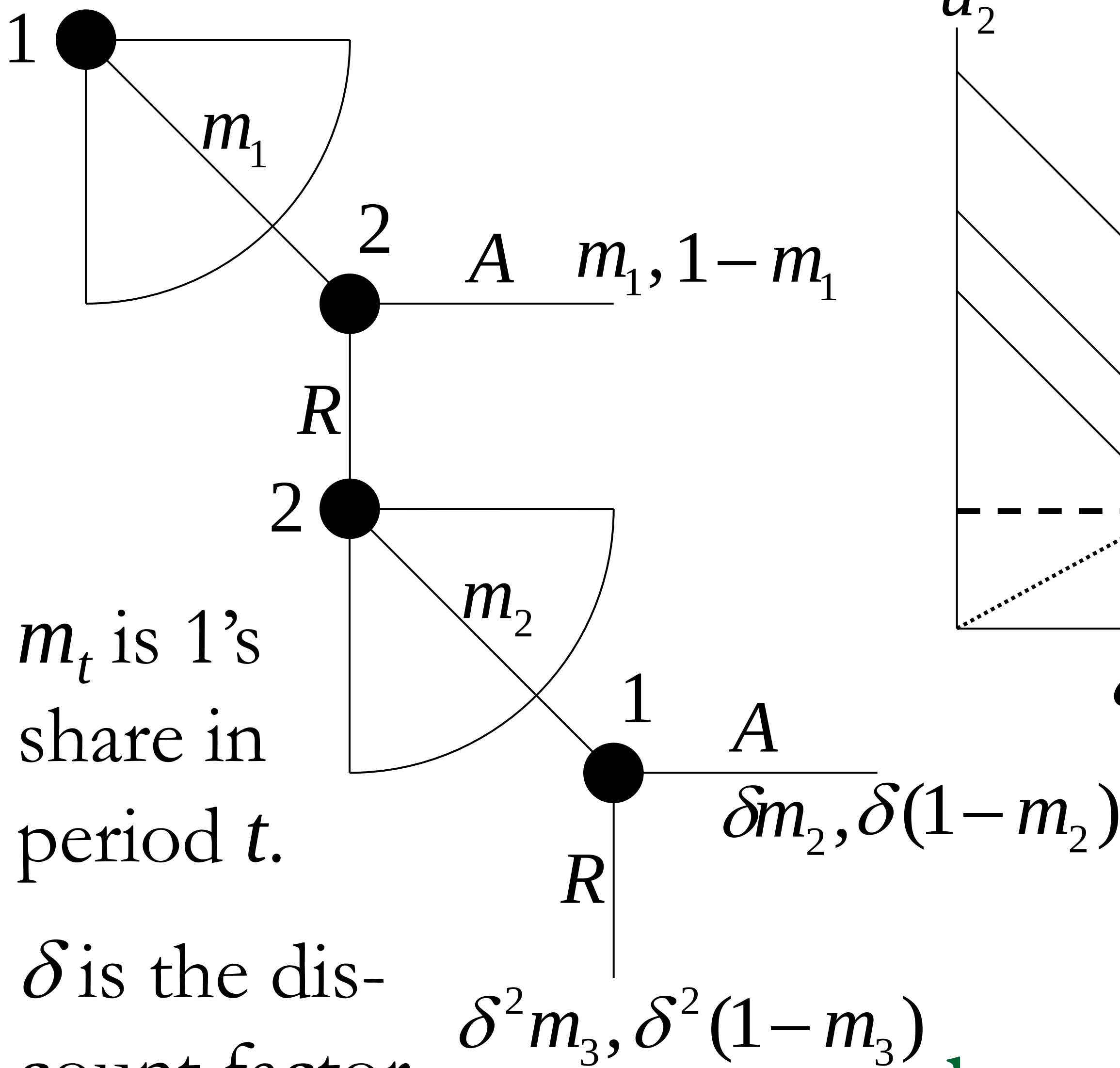
Three-period alternating offer

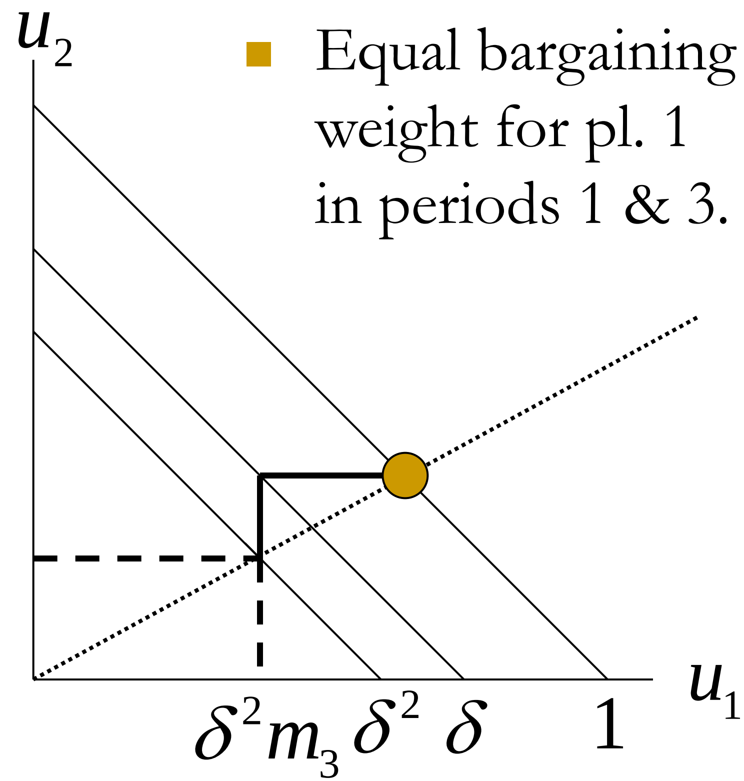


m_t is 1's share in period t.

δ is the discount factor







- Equal bargaining weight for pl. 1 in periods 1 & 3.

m_t is 1's share in period t.

δ is the discount factor

Rubinstein's bargaining model

Solution of Rubinstein's bargaining model with an infinite time horizon

Unique subgame perfect Nash equilibrium:

$$1 \text{ demands } m_t = \frac{1}{1+\delta}$$

$$1 \text{ accepts if and only if } m_t \geq \frac{\delta}{1+\delta}$$

$$2 \text{ offers } m_t = \frac{\delta}{1+\delta} = 1 - \frac{1}{1+\delta}$$

$$2 \text{ accepts if and only if } (1 - m_t) \geq \frac{\delta}{1+\delta} = 1 - \frac{1}{1+\delta}$$

1's proposal at time $t = 1$ is accepted.

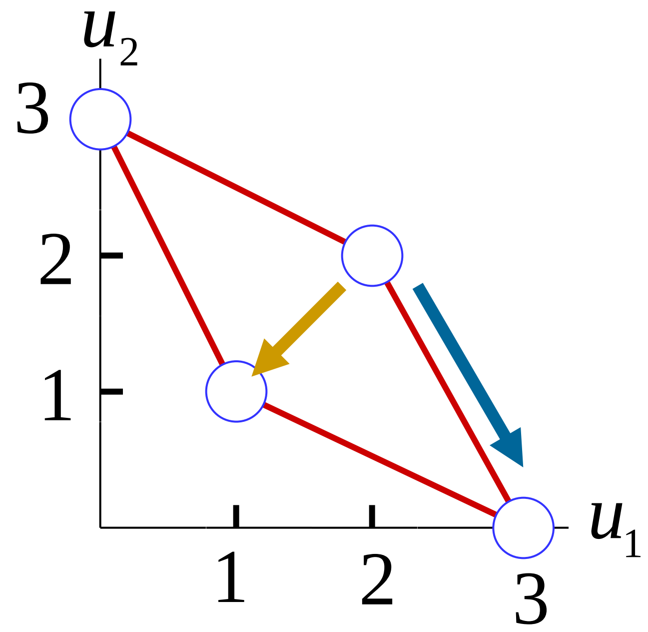
1st mover advantage: 1's share is larger than 2's.

If disc. factors are different: It pays to be patient.

Repeated games with an infinite time horizon

Prisoners' dilemma

	D	C
D	1, 1	3, 0
C	0, 3	2, 2



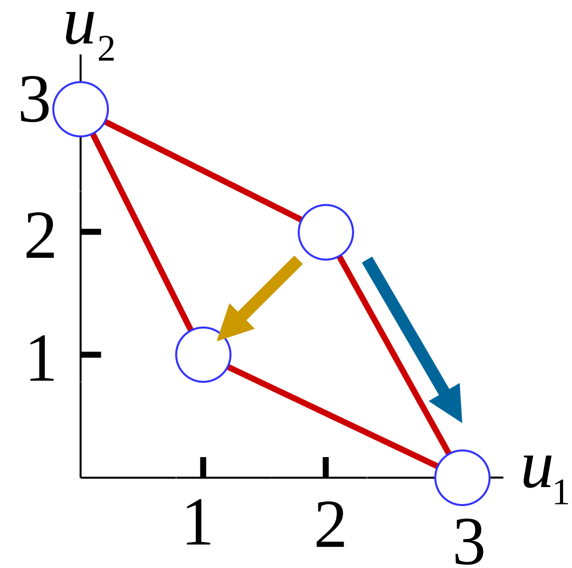
Can repetition discipline the players to cooperate?

Deviating yields a short run gain.

Deviating yields a loss of reputation that undermines future cooperation.

Yes, if gain now ≤ PV of future loss

Cooperation in infinitely rep. Prisoners' Dil.



"Trigger strategy":

Play D if D has been used earlier;

otherwise play C. (Start with C.)

If cooperation breaks down, it will never be restarted.

Subgame perfect Nash equilibrium (NE in all subgames)?

If cooperation has broken down: NE in the subgames.

If cooperation has not broken down:

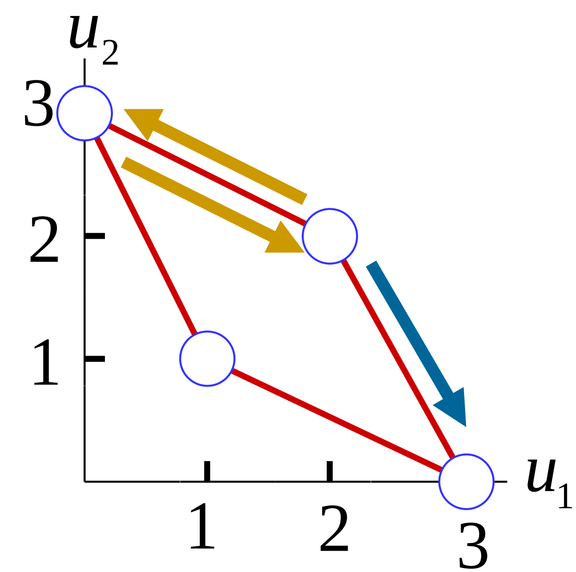
Short-run gain $\leq$ PV of long-run loss

$$\Rightarrow \quad 3-2 \leq \frac{\delta}{1-\delta}(2-1) \quad \Rightarrow \quad 1 \leq \delta 2 \quad \Rightarrow \quad \delta \geq \frac{1}{2}$$

"Getting Even"

Play C, unless "permitted" to play D.

"Permitted" to play D if the opponent played D w/o "perm" in previous period.



If 1 deviates, then 1 is punished in the next period.

Short-run gain $\leq$ PV of loss in next period

If cooperation has not broken down:

$$\Rightarrow \quad 3-2 \leq \delta(2-0) \quad \Rightarrow \quad 1 \leq \delta 2 \quad \Rightarrow \quad \delta \geq \frac{1}{2}$$

If cooperation has broken down:

$$\Rightarrow \quad 1-0 \leq \delta(2-0) \quad \Rightarrow \quad 1 \leq \delta 2 \quad \Rightarrow \quad \delta \geq \frac{1}{2}$$

Repeated games– Economic lessons

Two sources of instability: 1) temptation for deviation in coop stage and 2) temptation to deviate when the player is being punished. With the numbers in previous example the two factors are equally strong.

Other numbers may change the relative force.

”Getting even” and ”trigger strategy” are ways of upholding cooperation. In previous example they happen to be equally stable. Usually they are not.

In examples, the will to punish the others was a non-issue. In many economic settings there is a cost, need to check that punishment is a NE.

Repeated games– Economic lessons

The results hinge on the game being infinitelly repeated. Finite PD:

- In last period, DD is only NE no matter of history.
- Therefore payoffs by actions in second to last period is indep of payoff in last period.
- Hence, in second to last period, DD is only NE
- Similar backward induction leads to DD in all periods.