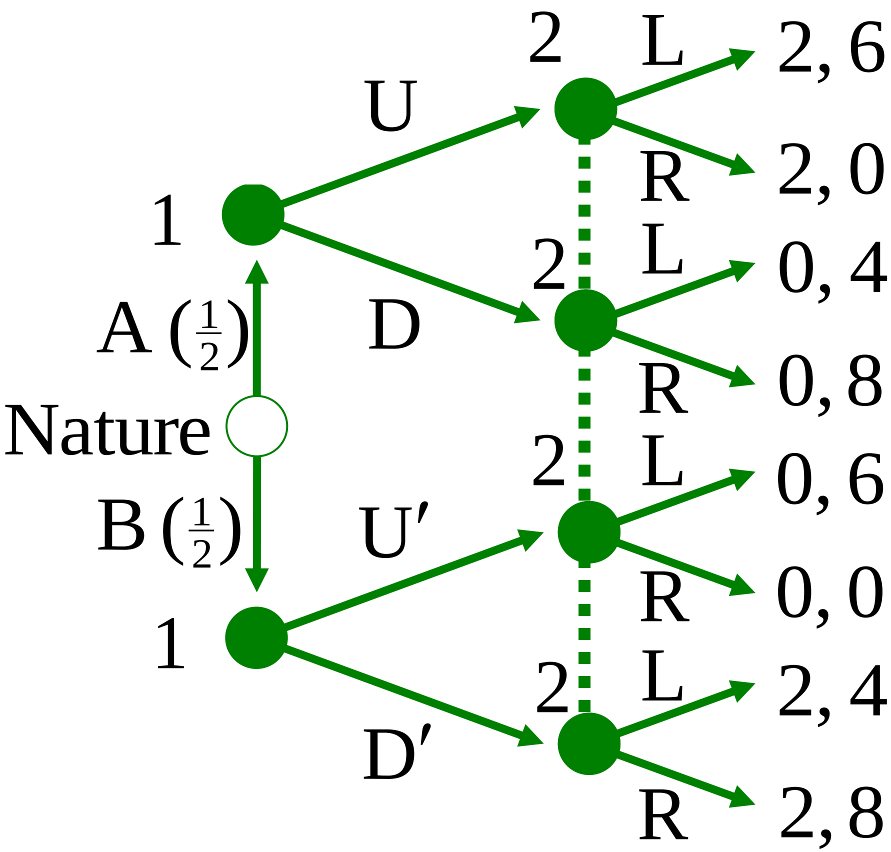
Incomplete information: Bayesian Nash equilibrium – knowing yourself but not your opponent.

Lectures in Game Theory

Fall 2012, Lecture 5

- **Incomplete information**: At least one player does not know who his opponents are.
- An illustrating example
- Definition of
  Bayesian games,
  Bayesian normal form,
  Bayesian Nash equilibrium
- **First-price sealed-bid auction** as an example.
- **Cournot competition** as an example

A Bayesian game



Either

1 \ 2	L	R
U	2, 6	2, 0
D	0, 4	0, 8

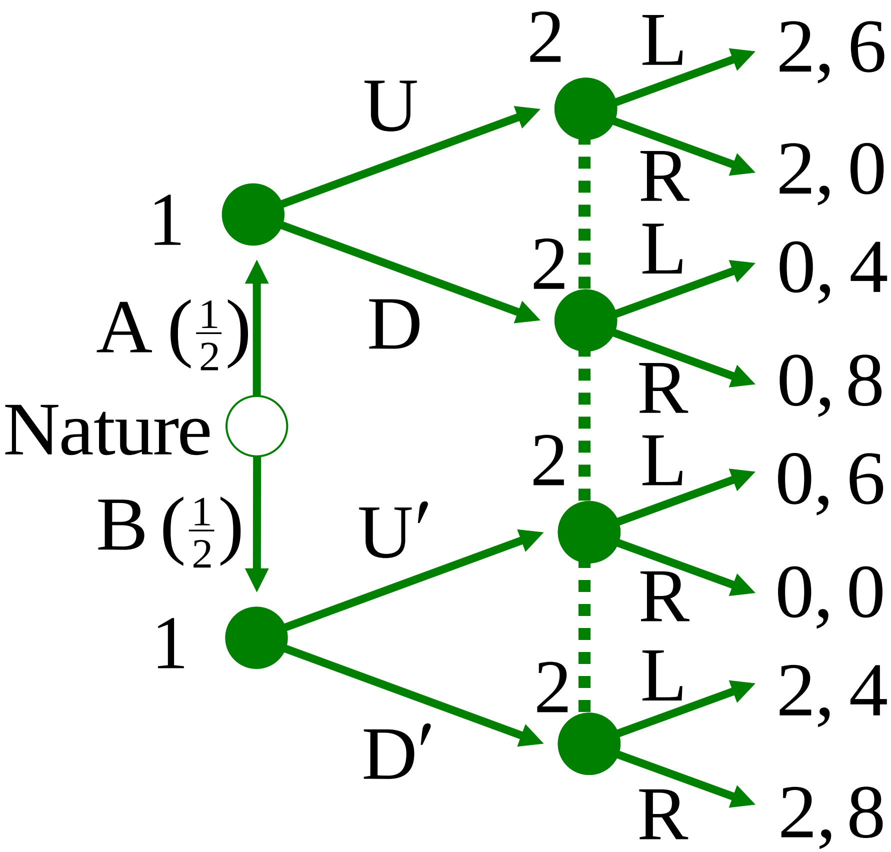
or

1 \ 2	L	R
U′	0, 6	0, 0
D′	2, 4	2, 8

is played, but only 1 knows which.

Note: Payoffs for 2 are same for a given strategy combination, regardless of 1's type.

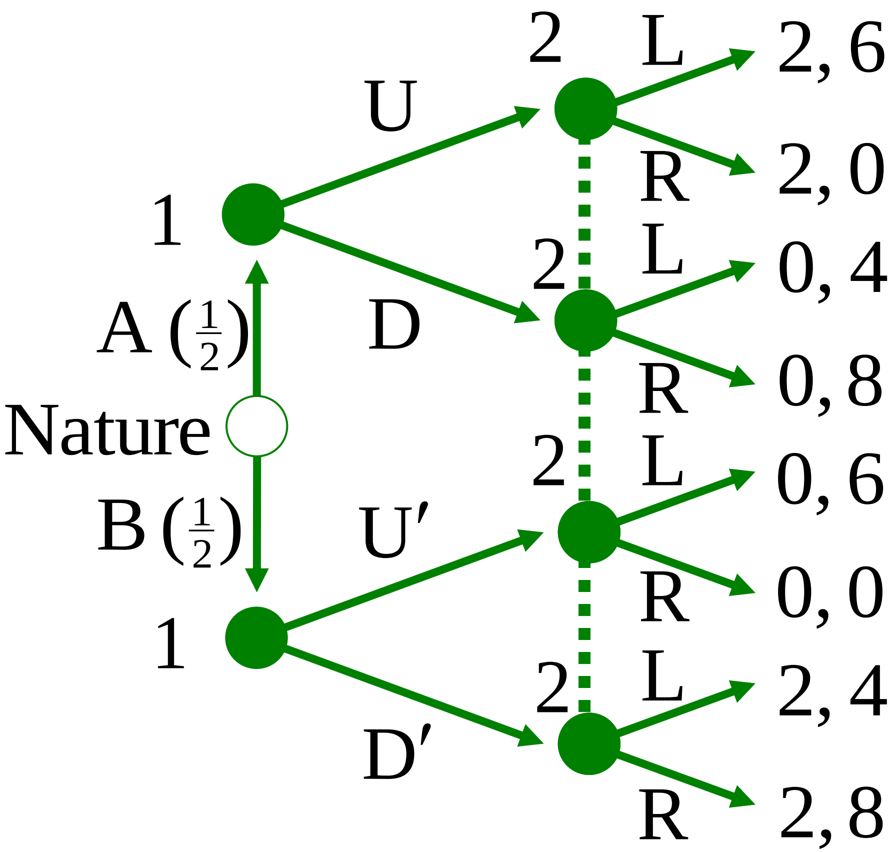
The Bayesian normal form (an *ex ante* perspective)



1 \ 2	L	R
UU′	1, 6	1, 0
UD′	2, 5	2, 4
DU′	0, 5	0, 4
DD′	1, 4	1, 8

Bayesian rationalizability and **Bayesian Nash equilibrium**

Treating different types as different players (an *ex post* perspective)



1A: U

1B \ 2	L	R
U′	2, 0, 6	2, 0, 0
D′	2, 2, 5	2, 2, 4

1A: D

1B \ 2	L	R
U′	0, 0, 5	0, 0, 4
D′	0, 2, 4	0, 2, 8

Equivalent way to determine **Bayesian Nash equilibrium**

A Bayesian game specifies

- *Players*: $\{1, \ldots, i, \ldots, n\}$
- For each player, an *action set*: A_i
- For each player, a *type set*: T_i
- A *probability distribution* over type profiles: p
- For each player, a *payoff function*: u_i

Bayesian game: $G = (A_1, \ldots, A_n; T_1, \ldots, T_n; p; u_1, \ldots, u_n)$

Player i's *type* $t_i \in T_i$ is private information.

Player i's *payoff* $u_i(a_1, \ldots, a_n; t_1, \ldots, t_n)$

depends on the action and type profiles.

Strategy

Definition : In the Bayesian game

$$G = (A_1, \ldots, A_n; T_1, \ldots, T_n; p; u_1, \ldots, u_n)$$

a *strategy* for player i is a function $s_i(\cdot)$ that,

for each type $t_i \in T_i$, specifies a feasible action $s_i(t_i)$.

The Bayesian normal form specifies

- *Players*: $\{1, \ldots, i, \ldots, n\}$
- For each player, the *strategy set*: S_i
- For each player, the *expected payoff function*

Definition: A *Bayesian Nash equilibrium* of a Bayesian game is a Nash equilibrium of the Bayesian normal form.

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English: A *Bayesian Nash equilibrium* is strategies for player 1 of type, player 1 of type 2,…
and player 2 of type 1, player 2 of type 2,…
….
player n of type 1, type 2,...
such that none of them would regret if they hear of the strategies of the other players of all types.

Note: A player needs to consider the strategies of herself had she been someone else.
Why? The other players do not know who I am.

1st price sealed bid auction w/private values

$$\textbf{Bid}: \quad b_i \in A_i = [0,1000],\ i = 1,2$$

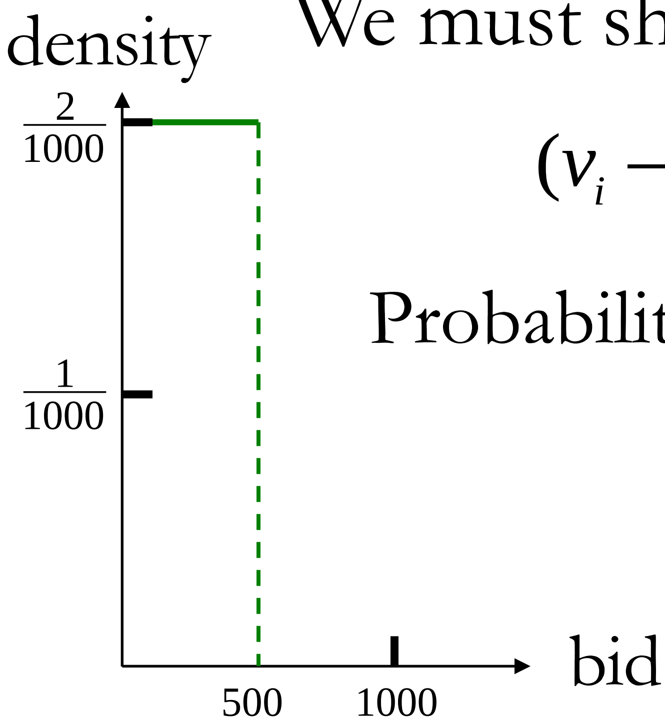
$\textbf{Valuations}: \quad v_i \in T_i = [0,1000],\ i = 1,2,$ are distributed independently and uniformly : For each $\boldsymbol{i}$, $\Pr(v_j \leq x) = \frac{x}{1000}$.

$$u_i(b_i, b_j; v_i) = \begin{cases} v_i - b_i & \text{if } \boldsymbol{b_i} \text{ is bigger than the other bid.} \\ 0 & \text{otherwise.} \end{cases}$$

One must bid less than true value in order to earn if one wins. This must be traded off against the fact that a lower bid reduces the probability for having a winning bid.

<u>Consider strategies of the following form</u>: $b_i(v_i) = \frac{1}{2} v_i$

A Bayesian Nash equil. in 1st price auction



We must show that $b_i(v_i) = \frac{1}{2} v_i$ maximizes

$$(v_i - b_i) \cdot \Pr(\text{Other bids less than } b_i)$$

Probability that other bids less than ***b_i***: $\frac{2}{1000} b_i$

$$\max_{b_i} (v_i - b_i) \cdot \frac{2}{1000} b_i$$

FOC yields: $b_i = \frac{1}{2} v_i$

This shows that it is a Bayesian Nash equil. that both bid half of true value.

- ***n*** bidder auction
- Cournot competition