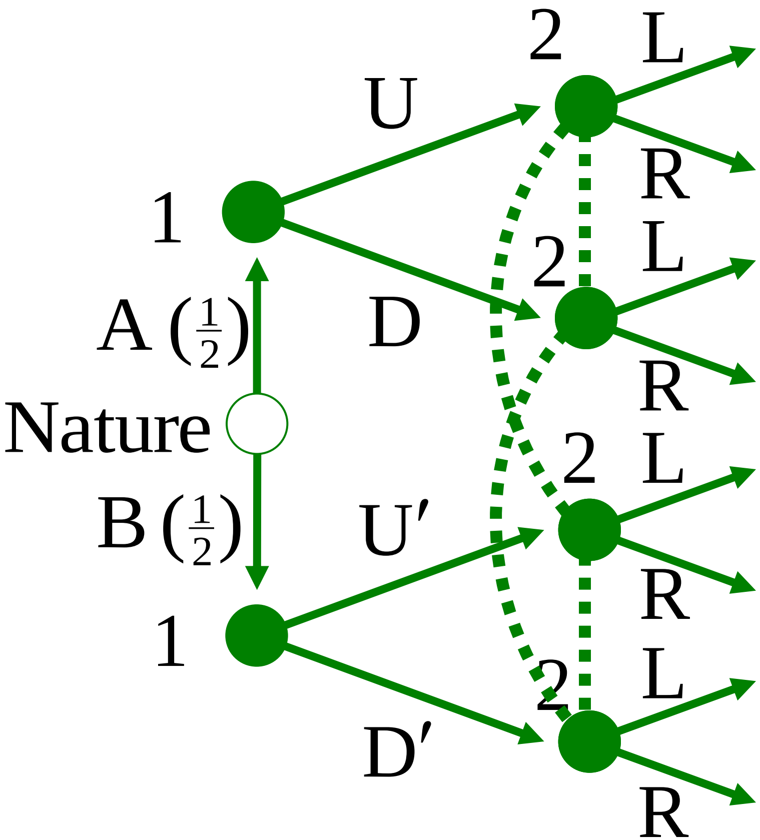
Incomplete information: Perfect Bayesian equilibrium

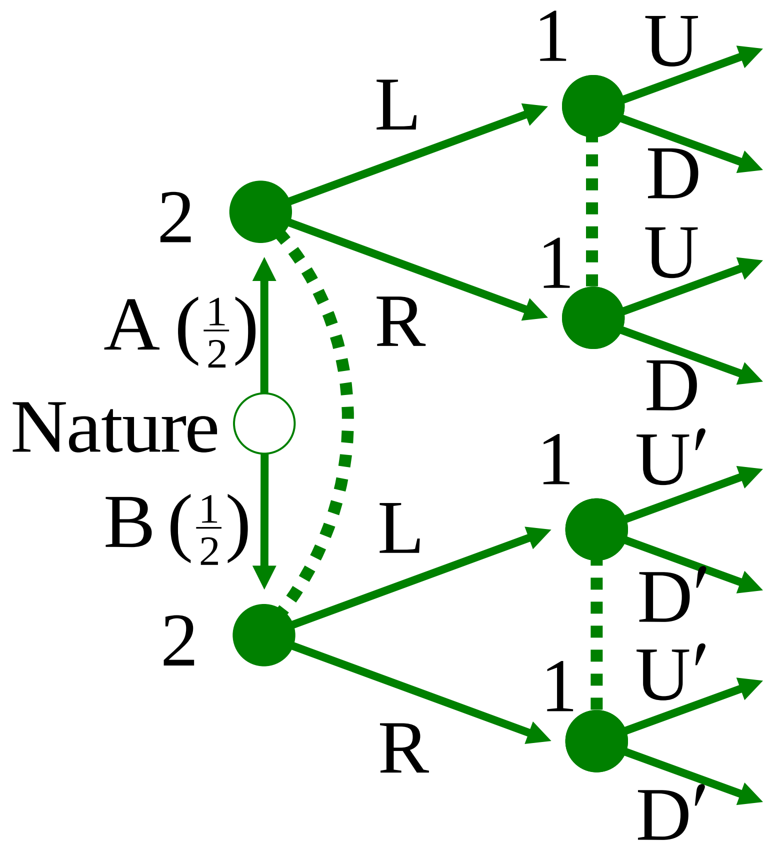
Lectures in Game Theory

Fall 2012, Lecture 6

A static Bayesian game

Equivalent representation:



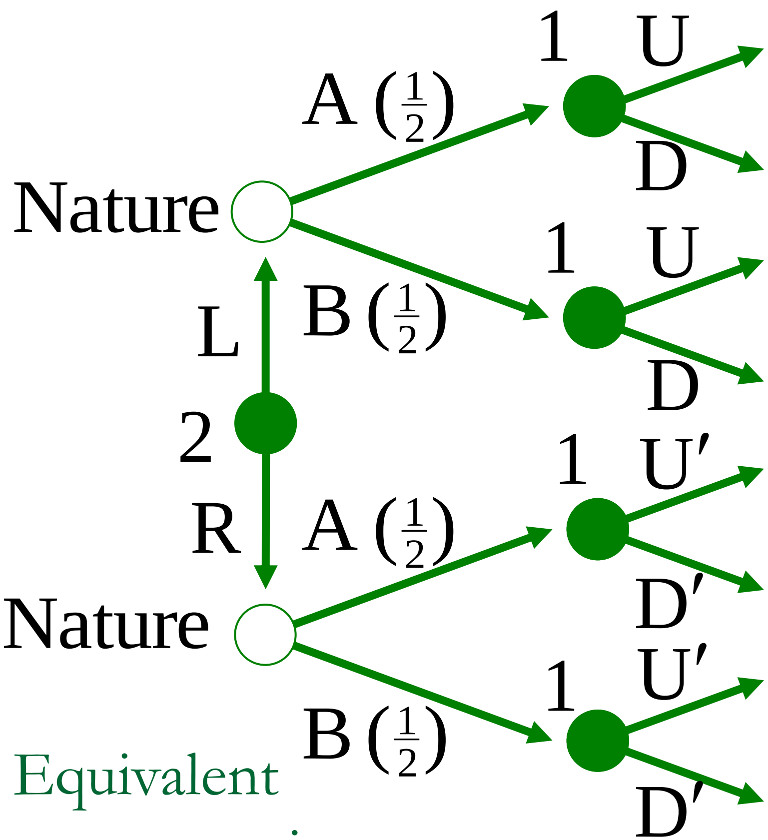


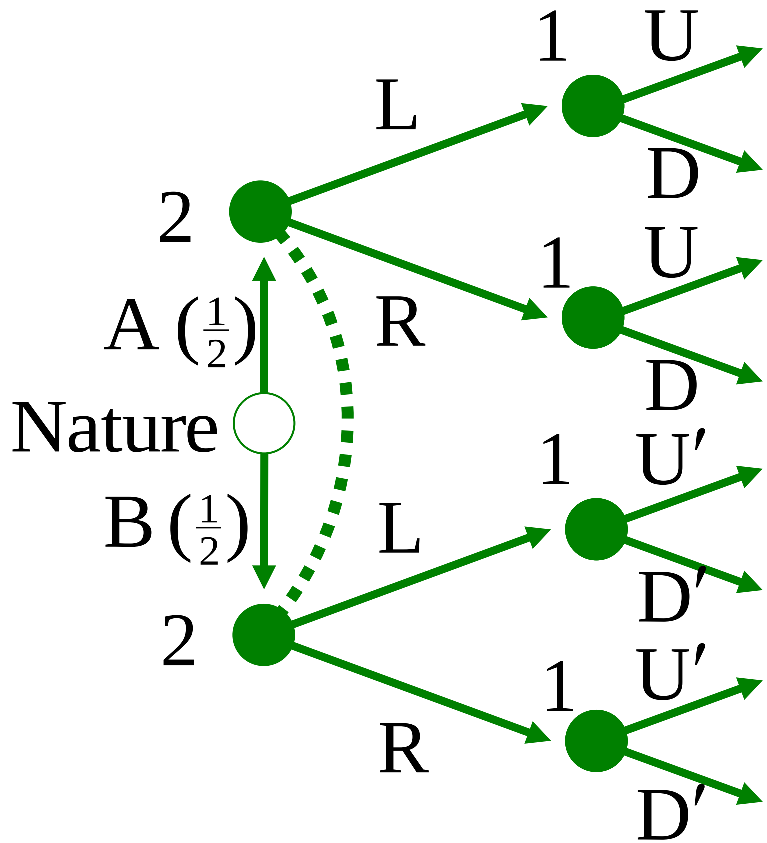
What if the uninformed gets to observe the informed choice?

What if the informed gets to observe the uninformed choice?

A dynamic Bayesian game: Screening

The informed gets to observe the uninformed choice.



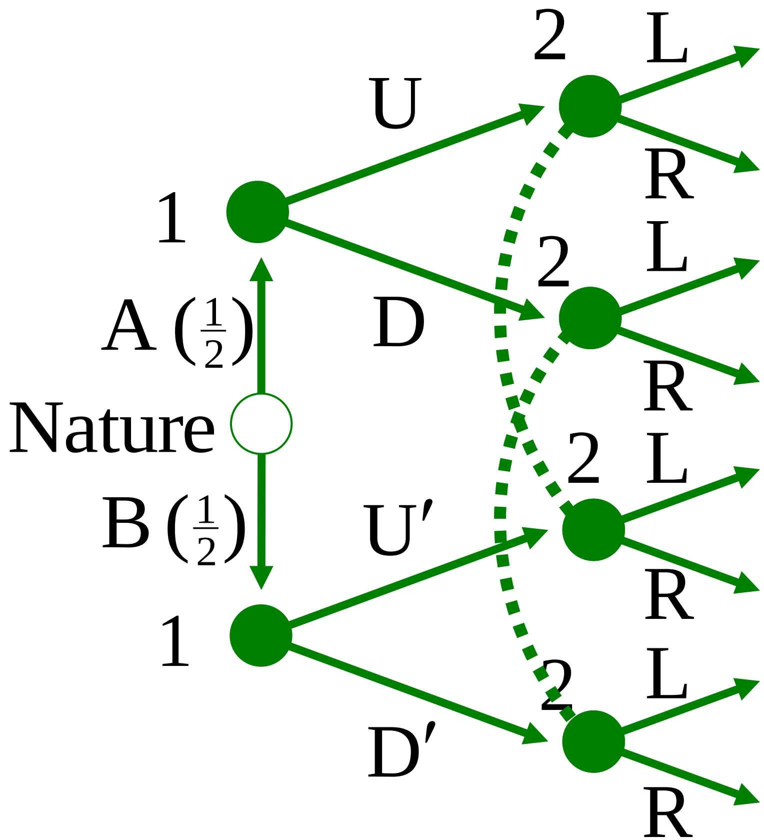


Equivalent representation:

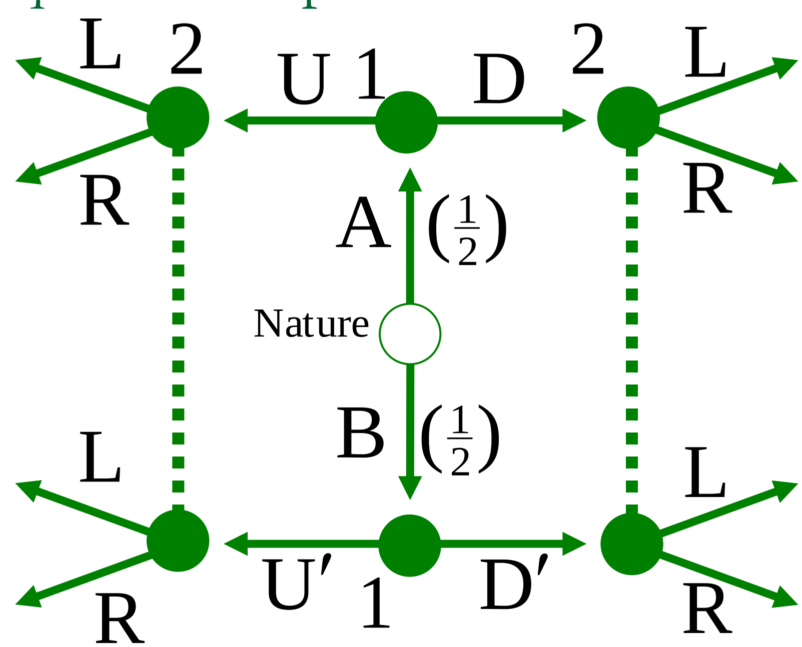
Use subgame perfect Nash equilibrium!

A dynamic Bayesian game: Signaling

The uninformed gets to observe the informed choice.



Equivalent representation:

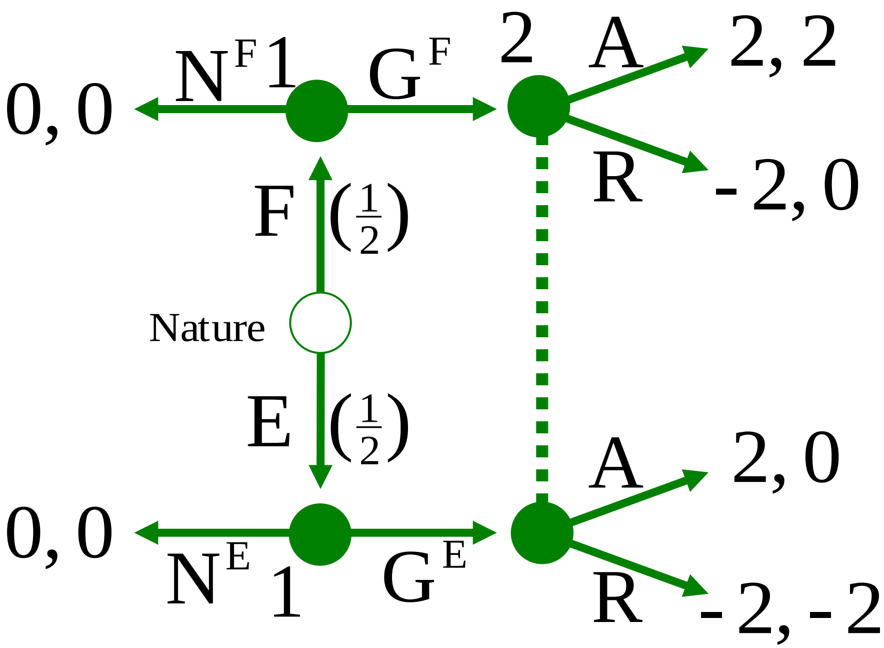


Requires a new equilibrium concept:

Perfect Bayesian equilibrium

Why?

Gift game, version 1

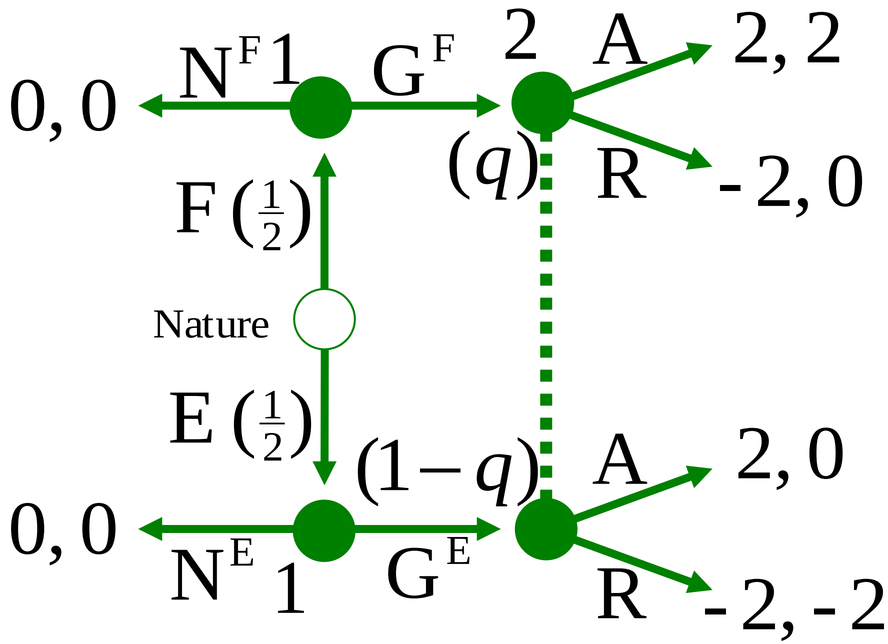


1 \ 2	A	R
$G^F G^E$	2, 1	-2, -1
$G^F N^E$	1, 1	-1, 0
$N^F G^E$	1, 0	-1, -1
$N^F N^E$	0, 0	0, 0

Are both Nash equilibria reasonable?

Note that subgame perfection does not help. Why?

Remedy 1: Conditional beliefs about types



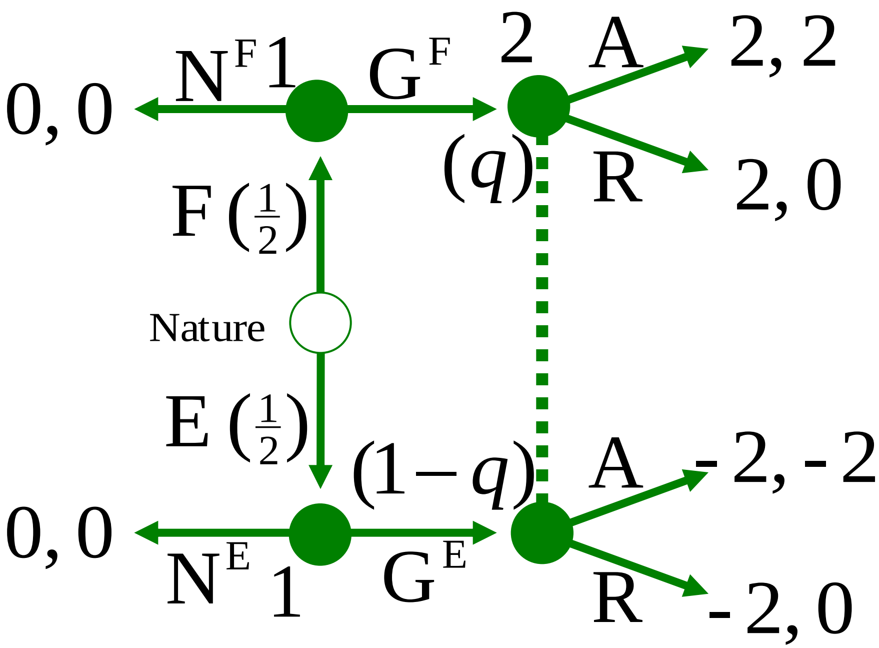
Let player 2 assign probabilities to the two types player 1: the ***belief*** of player 2.

What will the players choose?

Remedy 2: Sequential rationality

Each player chooses rationally at all information sets, given his belief and the opponent's strategy.

Gift game, version 2

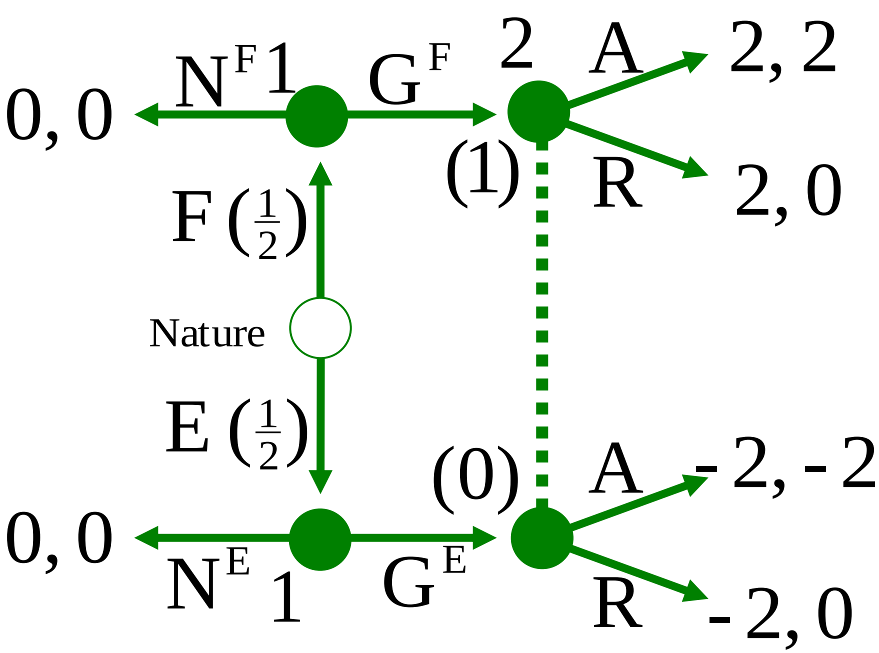


1 \ 2	A	R
$G^F G^E$	0, 0	0, 0
$G^F N^E$	1, 1	1, 0
$N^F G^E$	-1, -1	-1, 0
$N^F N^E$	0, 0	0, 0

If $q < ½$, then player 2 will choose R.

If so, the outcome is not a Nash equil. outcome.

Remedy 3: Consistency of beliefs



Player 2 should find his belief by means of Bayes' rule, when-ever possible.

$$q = \Pr[F \mid G] \equiv \frac{\Pr[G \mid F]\Pr[F]}{\Pr[G]}$$

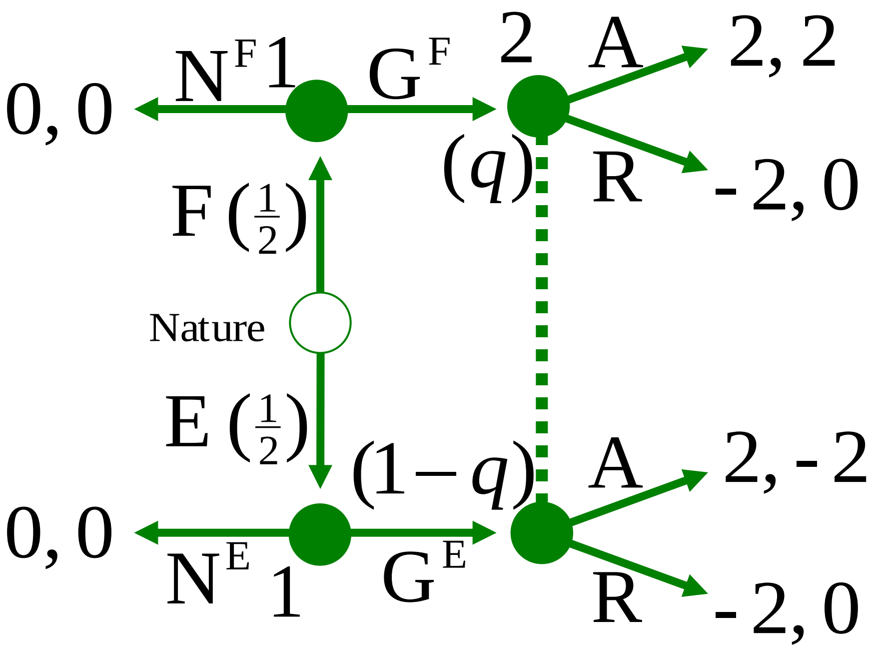
An example of a *separating equilibrium*.

An equilibrium is ***separating*** if the types of a player behave differently.

Gift game, version 3

If $q < ½$, then player 2 will choose R.

Bayes' rule cannot be used.



1 \ 2	A	R
G^FG^E	2, 0	-2, 0
G^FN^E	1, 1	-1, 0
N^FG^E	1, -1	-1, 0
N^FN^E	0, 0	0, 0

An example of a *pooling equilibrium*. An equilibrium is ***pooling*** if the types behave the same.

Perfect Bayesian equilibrium

- **Definition:** Consider a strategy profile for the players, as well as beliefs over the nodes at all information sets. These are called a ***perfect Bayesian equilibrium*** (PBE) if:

  (a) each player's strategy specifies optimal actions given his beliefs and the strategies of the other players.

  (b) the beliefs are consistent with Bayes' rule whenever possible.

Algorithm for finding perfect Bayesian equilibria in a signaling game:

- posit a strategy for player 1 (either pooling or separating),
- calculate restrictions on conditional beliefs,
- calculate optimal actions for player 2 given his beliefs,
- check whether player 1’s strategy is a best response to player 2’s strategy.

Applying the algorithm in a signaling game

Player 1 has four pure strategies.

PBE w/(LL′)? **YES**

$[(LL'), (DU'), q, r = 1/2]$

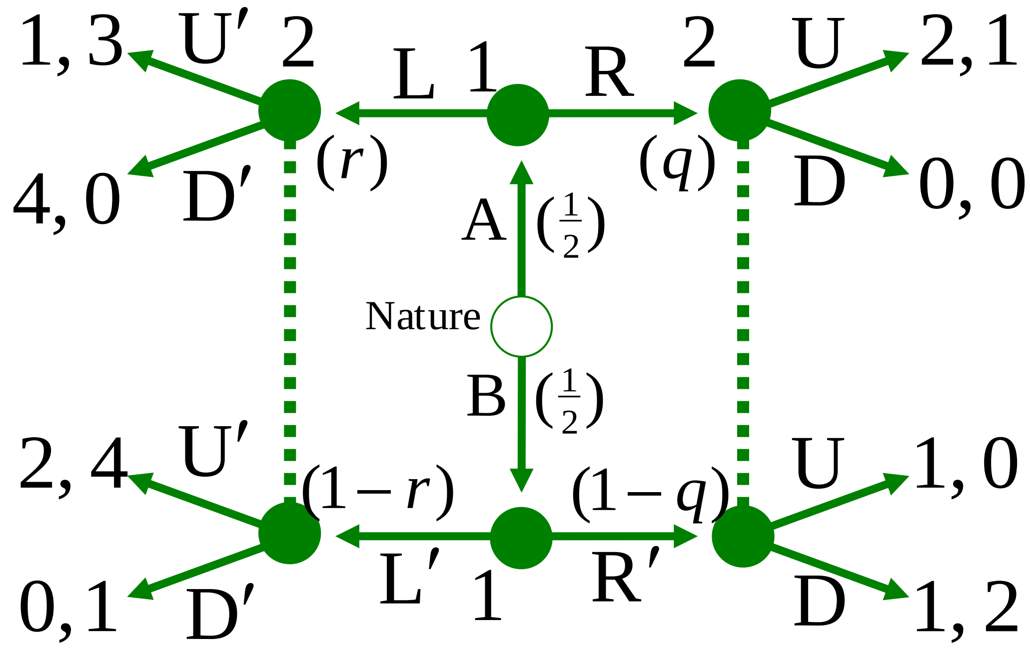
where $q \leq 2/3$.

PBE w/(RR′)? **NO**

PBE w/(LR′)? **NO**

PBE w/(RL′)? **YES**

$[(RL'), (UU'), q = 1, r = 0]$



$[(RL'), (UU'), q = 1, r = 0]$ is a separating equilibrium.

$[(LL'), (DU'), q, r = 1/2]$ is a pooling equilibrium.