Problems for the tenth seminar: Incomplete information in dynamic games

ECON3200 Microeconomics and game theory — Fall semester 2012

Solutions to the problems will be presented 21–23 November 2012.

Problem 1 *(Screening and signaling)*

Consider again the strategic situation described in Problem 2 of the set for the ninth seminar, where only player 1 knows which game is being played, while player 2 thinks that the two games are equally likely.

(a) (Screening) Assume now that player 2 acts before player 1, and that 2's choice can be observed by 1 before he makes his choice. Show that there is a unique subgame perfect Nash equilibrium.

(b) (Signaling) Assume now that player 1 acts before player 2, and that 1's choice can be observed by 2 before she makes her choice. Show that there is a unique separating perfect Bayesian equilibrium. (Is there a pooling equilibrium?)

Problem 2 *(Simultaneous moves; Nash equilibrium)*

You and a friend are in a restaurant, and the owner offers both of you an 8-slice pizza under the following condition. Each of you must simultaneously announce how many slices you would like; that is, each player $i \in \{1, 2\}$ names his/her desired amount of pizza, $0 \leq s_i \leq 8$. If $s_1 + s_2 \leq 8$, then the players get their demands (and the owner eats any leftover slices). If $s_1 + s_2 > 8$, then the players get nothing. Assume that you each care only about how much pizza you individually consume, preferring more pizza to less.

(a) What is (are) each player's best response(s) for each of the possible demands for his/her opponent?

(b) Find all the pure-strategy Nash equilibria.

Problem 3 *(Sequential moves; Nash and Subgame perfect Nash equilibrium)*

Consider the situation of Problem 2, but assume now that player 1 makes her demand before player 2 makes his demand. Player 2 observes player 1's demand before making his choice.

(a) Explain what a strategy is for player 2 in this game with sequential moves.

(b) Find all the pure-strategy Nash equilibrium outcomes.

(c) Find all the pure-strategy subgame perfect equilibria.

Problem 4 *(Sequential moves and incomplete information; Perfect Bayesian equilibrium)*

Consider the situation of Problem 3, but assume now in addition that the pizza comes in 5 different sizes, each with x slices, where $x \in \{4, 6, 8, 10, 12\}$. Player 1 observes x before making her demand, while players 2 only observes player 1's demand, but not x, before having to make his own demand. Before observing player 1's demand, player 2 thinks that the 5 different pizza sizes are equally likely, but he may infer something from her demand.

(a) Explain what a strategy is for player 1 in this game of incomplete information.

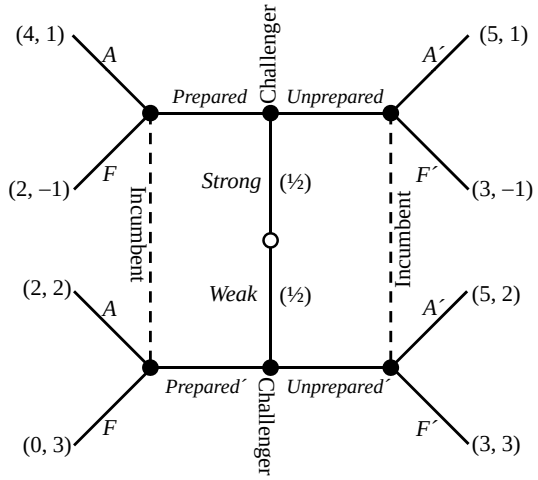
(b) Show that the following strategy for player 1 can be part of a perfect Bayesian equilibrium: $s_1(4) = 2$, $s_1(6) = 3$, $s_1(8) = 4$, $s_1(10) = 5$, $s_1(12) = 11$. Specify both player 2's strategy and player 2's beliefs.

(c) Are there other perfect Bayesian equilibria in this game?

Problem 5 *(Challenging an incumbent)*

Consider a market where there is an incumbent firm and a challenger. The challenger is *strong* with probability $\frac{1}{2}$ and *weak* with probability $\frac{1}{2}$; it knows its type, but the incumbent does not. The challenger may either *prepare* itself for battle or remain *unprepared*. The incumbent observes the challenger's

preparedness, but not its type, and chooses whether to *fight* (F) or *acquiesce* (A). The extensive form and the payoffs are given by the following figure. The challenger's payoff is listed first, the incumbent's second.



(a) What are the (pure) strategies for the challenger?

(b) Why is there no perfect Bayesian equilibrium where the weak challenger chooses *Prepared′* ?

(c) Show that there is a perfect Bayesian equilibrium where the strong challenger chooses *Prepared* and the weak challenger chooses *Unprepared′*. What do we call such an equilibrium?

(d) Show that there is a perfect Bayesian equilibrium where the strong challenger chooses *Unprepared* and the weak challenger chooses *Unprepared′*. What do we call such an equilibrium?