Problems for the eighth seminar: Applications of dynamic games

ECON3200 Microeconomics and game theory — Fall semester 2012

Solutions to the problems will be presented 7–9 November 2012.

Problem 1 *("Rotten kid theorem")*

A child's action a (a number) affects both her own private income $c(a)$ and her parents' income $p(a)$; for all values of a we have that $c(a) < p(a)$. The child is selfish: she cares only about the amount of money she has. Her loving parents care both about how much money they have and how much their child has. Specifically, model the parents as a single individual whose preferences are represented by a payoff equal to the smaller of the amount of money they have and the amount of money the child has. The parents may transfer money to the child. First the child takes an action, then the parents decide how much money to transfer. Model this situation as an extensive game and show that in subgame perfect equilibrium the child takes an action that maximizes the sum of her private income and her parents' income. In particular, the child's action does not maximize her own private income.

Problem 2 *(Firm-union bargaining)*

A firm's output is $L(100 - L)$ when it uses $L \leq 50$ units of labor, and 2500 when it uses $L \geq 50$ units of labor. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative number w), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number L of workers to employ (which you should take to be a continuous variable, not an integer); if it rejects the demand, no production takes place ($L = 0$). The firm's preferences are represented by its profits, the union's preferences are represented by the value of wL.

(a) Formulate this situation as an extensive game with perfect information.

(b) Find the subgame perfect equilibrium (equilibria?) of the game.

(c) Is there an outcome of the game that both parties prefer to any subgame perfect equilibrium outcome?

(d) Find a Nash equilibrium for which the outcome differs from any subgame perfect equilbrium outcome.

Problem 3 *(Finitely repeated game)*

Watson Exercise 22.4

Problem 4 *(Infinitely repeated game)*

Consider the following normal form game previously analyzed in Problem 4 of the set for the seventh seminar.

	L	C	R
U	$1,1$	$4,0$	$5,0$
M	$0,4$	$3,3$	$6,2$
D	$0,5$	$2,6$	$5,5$

Assume now that this game is repeated infinitely many times. Assume furthermore that the players in each round can observe choices made in earlier rounds and that their payoff is the sum of payoffs discounted by the discount factor δ (where $0 < \delta < 1$). Show that there exists a subgame perfect Nash equilibrium leading to the outcome path

$$(D, R), (D, R), (D, R), (D, R), \ldots$$

if $\delta \geq 1/5$.

Problem 5 *(Infinitely repeated price competition)*

Watson Exercise 23.1, with the following additional question, to be answered before part (a):

What is the lowest price $\underline{p}$ in a Nash equilibrium if the game is played only once? And what is the profits of the firms in a Nash equilibrium in this case?

In question c, rephrase to "what does this say about the stability of a cartel?"