Part 1: Solutions and Guidelines

December 3, 2015

1 FIRM [max 20 points]

A. [max 4 points]

[2 points if the MRTS is well-defined; only 1 point if well-defined, but computation mistake; only 1 point if the definition mixes the indeces/computation is wrong]

The marginal rate of technical subsitution between inputs 1 and 2 is:

$$MRTS_{12} = \frac{\frac{\partial \phi}{\partial z_2}}{\frac{\partial \phi}{\partial z_1}} = \frac{\frac{1}{2}\frac{1}{z_2}}{\frac{1}{4}\frac{1}{z_1}} = 2\frac{z_1}{z_2}.$$

Alternative answer:

$$MRTS_{21} = \frac{\frac{\partial \phi}{\partial z_1}}{\frac{\partial \phi}{\partial z_2}} = \frac{\frac{1}{4}\frac{1}{z_1}}{\frac{1}{2}\frac{1}{z_2}} = \frac{1}{2}\frac{z_2}{z_1}.$$

[2 points if the marginal rate of substitution is correctly evaluated at $(z_1, z_2) = (1, 2)$; no deduction if the answer is correct, conditional on a computational mistake in the previous; only 1 point if there is a new computational mistake]

$$MRTS_{12}|_{(z_1,z_2)=(1,2)} = 2\frac{z_1}{z_2}\bigg|_{(z_1,z_2)=(1,2)} = \frac{2}{1}\frac{1}{2} = 1.$$

alternative answer:

$$MRTS_{21}|_{(z_1,z_2)=(1,2)} = \frac{1}{2}\frac{z_2}{z_1}\bigg|_{(z_1,z_2)=(1,2)} = \frac{1}{2}\frac{2}{1} = 1.$$

B. [max 12 points]

[2 point if the cost minimization problem is well-defined] The firm cost minimization problem is:

$$\begin{array}{cc} \min_{z_1,z_2} & w_1 z_1 + w_2 z_2 \\ s.t. & q \le \phi(z) = \frac{1}{4}\ln z_1 + \frac{1}{2}\ln z_2 \end{array}$$

[1 point if the following point is made explicit] Since the production function is strictly increasing in inputs quantities, we can substitute the inequality by an equality:

$$\begin{array}{cc} \min_{z_1,z_2} & w_1 z_1 + w_2 z_2 \\ s.t. & q = \frac{1}{4}\ln z_1 + \frac{1}{2}\ln z_2 \end{array}$$

[1 point if the following point is made explicit] Since quantity produced is -∞ when an input goes to 0, z_1 and z_2 are strictly positive and no KKT conditions are needed.

[2 point if the Lagrangean is correctly stated] The Lagrangean for the cost minimization problem is:

$$\mathcal{L}(z_1, z_2, \lambda) = w_1 z_1 + w_2 z_2 + \lambda \left(q - \frac{1}{4}\ln z_1 + \frac{1}{2}\ln z_2 \right)$$

[1 point if the FOCs are well-derived] The FOCs are (I skip the stars marking optimally set variables):

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial z_1} = 0 & \Rightarrow \quad w_1 = \lambda \frac{1}{4}\frac{1}{z_1} \\ \frac{\partial \mathcal{L}}{\partial z_2} = 0 & \Rightarrow \quad w_2 = \lambda \frac{1}{2}\frac{1}{z_2} \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 & \Rightarrow \quad q = \frac{1}{4}\ln z_1 + \frac{1}{2}\ln z_2 \end{cases}$$

Solving. Divide the first by the second condition gives:

$$\frac{w_1}{w_2} = \frac{1}{2}\frac{z_2}{z_1} \quad or \quad z_2 = 2\frac{w_1}{w_2} z_1$$

Substituting in the last FOC, gives:

$$q = \frac{1}{4}\ln z_1 + \frac{1}{2}\ln\left(2\frac{w_1}{w_2} z_1\right)$$

$$q = \frac{1}{4}\ln z_1 + \frac{1}{2}\ln z_1 + \frac{1}{2}\ln 2\frac{w_1}{w_2}$$

$$q = \frac{3}{4}\ln z_1 + \frac{1}{2}\ln 2\frac{w_1}{w_2}$$

$$\frac{3}{4}\ln z_1 = q - \frac{1}{2}\ln 2\frac{w_1}{w_2}$$

$$\ln z_1 = \frac{4}{3}q - \frac{2}{3}\ln 2\frac{w_1}{w_2}$$

$$z_1 = e^{\frac{4}{3}q}\left(2\frac{w_1}{w_2}\right)^{-\frac{2}{3}}$$

[1 point if the conditional demand of input 1 is well-derived]

$$H_1(w_1, w_2, q) = z_1 = \left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q}.$$

Substituting this back in the last FOC, gives the demand for input 2:

$$q = \frac{1}{4}\ln\left[\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q}\right] + \frac{1}{2}\ln z_2$$

$$\frac{1}{2}\ln z_2 = q - \frac{1}{4}\ln\left[\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q}\right]$$

$$\ln z_2 = 2q - \frac{1}{2}\ln\left[\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q}\right]$$

$$z_2 = e^{2q}\left[\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q}\right]^{-\frac{1}{2}}$$

$$z_2 = e^{\left(2-\frac{2}{3}\right)q}\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{-\frac{1}{3}}$$

[1 point if the conditional demand of input 2 is well-derived]

$$H_2(w_1, w_2, q) = z_2 = \left(2\frac{w_1}{w_2}\right)^{\frac{1}{3}} e^{\frac{4}{3}q}$$

[2 points if the definition of cost function is correctly stated]

The cost function of the firm is found by multiplying the input demands by prices and summing up:

$$C(w_1, w_2, q) = w_1 H_1(w_1, w_2, q) + w_2 H_2(w_1, w_2, q)$$

$$C(w_1, w_2, q) = w_1\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q} + w_2\left(2\frac{w_1}{w_2}\right)^{\frac{1}{3}} e^{\frac{4}{3}q}$$

$$C(w_1, w_2, q) = \left(w_1\left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} + w_2\left(2\frac{w_1}{w_2}\right)^{\frac{1}{3}}\right) e^{\frac{4}{3}q}$$

$$C(w_1, w_2, q) = \left(\left(\frac{1}{2}\right)^{\frac{2}{3}} w_1^{1-\frac{2}{3}} w_2^{\frac{2}{3}} + (2)^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{1-\frac{1}{3}}\right) e^{\frac{4}{3}q}$$

$$C(w_1, w_2, q) = \left(2^{-\frac{2}{3}} + 2^{\frac{1}{3}}\right) w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} e^{\frac{4}{3}q}$$

$$C(w_1, w_2, q) = 2^{-\frac{2}{3}}(1+2)\, w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} e^{\frac{4}{3}q}$$

[1 point if the cost function is correctly obtained]

$$C(w_1, w_2, q) = 2^{-\frac{2}{3}} 3 w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} e^{\frac{4}{3}q}$$

C. [max 4 points]

[2 points if the Shephard Lemma is correctly stated] The Shephard Lemma requires that:

$$\frac{\partial C(w_1, w_2, q)}{\partial w_1} = H_1(w_1, w_2, q)$$

$$\frac{\partial C(w_1, w_2, q)}{\partial w_2} = H_2(w_1, w_2, q)$$

[1 point for showing that the derivative of the cost function wrt w_1 is the conditional demand of input 1] For the first derivative:

$$\frac{\partial}{\partial w_1} 2^{-\frac{2}{3}} 3 w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} e^{\frac{4}{3}q} = 2^{-\frac{2}{3}} 3 \frac{1}{3} w_1^{\frac{1}{3}-1} w_2^{\frac{2}{3}} e^{\frac{4}{3}q} = 2^{-\frac{2}{3}} w_1^{-\frac{2}{3}} w_2^{\frac{2}{3}} e^{\frac{4}{3}q} =$$

$$= \left(\frac{1}{2}\frac{w_2}{w_1}\right)^{\frac{2}{3}} e^{\frac{4}{3}q} = H_1(w_1, w_2, q)$$

[1 point for showing that the derivative of the cost function wrt w_2 is the conditional demand of input 2] Similarly, for the second:

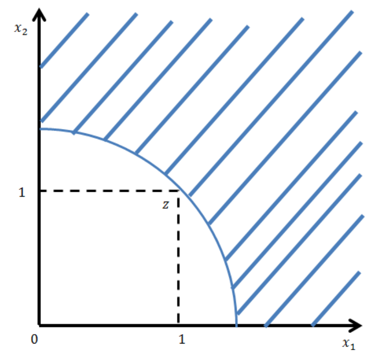
$$\frac{\partial}{\partial w_2} 2^{-\frac{2}{3}} 3 w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} e^{\frac{4}{3}q} = 2^{-\frac{2}{3}} 3 \frac{2}{3} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}-1} e^{\frac{4}{3}q} = 2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{-\frac{1}{3}} e^{\frac{4}{3}q} =$$

$$= \left(2\frac{w_1}{w_2}\right)^{\frac{1}{3}} e^{\frac{4}{3}q} = H_2(w_1, w_2, q)$$

2 CONSUMER [max 20 points]

D. [max 4 points]

[It is not too important that they know how to draw a portion of a circle, but that they understand the meaning of the concepts; so a certain degree of imprecision in the graph could still give 1 or 2 points if the student seems to understand the idea of no-worse-than-z-set] The no-worse-than-z-set for $z = (1, 1)$ is drawn as the shaded area in the following graph:



E. [max 3 points]

[1 point for correctly stating the property; 2 points for correctly checking the property] The no-better-than-z-set is the set below (including) the curve through z. As any bundle x is either in the first or the second set, preferences are complete.

Alternative answer: take any two bundles $x, z \in \mathbb{R}^2_+$, either $x_1^2+x_2^2 \geq z_1^2+z_2^2$, or $z_1^2 + z_2^2 \geq x_1^2 + x_2^2$, or $x_1^2 + x_2^2 = z_1^2 + z_2^2$. Thus preferences are complete.

F. [max 2 points]

[1 point for correctly stating the property; 1 point for correctly checking the property] Preferences are continuous since the no-worse-than-z-set and the no-better-than-z-set are closed, i.e. include their boundaries.

Alternative answer: take two sequences of bundles $\{x^n\}$ and $\{z\}^n$, the first converging to $\bar{x}$ as $n \to \infty$, the second converging to $\bar{z}$. Assume that for each $n \in \mathbb{N}$, $x^n \succsim z^n$. Then continuity requires that $\bar{x} \succsim \bar{z}$. By definition, for each $n \in \mathbb{N}$, $\left(x_1^n\right)^2 + \left(x_2^n\right)^2 \geq \left(z_1^n\right)^2 + \left(z_2^n\right)^2$. As the weak inequality is preserved under limit operation, also $\bar{x}_1^2 + \bar{x}_2^2 \geq \bar{z}_1^2 + \bar{z}_2^2$ holds and, by definition, $\bar{x} \succsim \bar{z}$.

G. [max 4 points]

[1 point for correctly stating the property; 3 points for correctly checking the property, only 1 if incomplete]

Graphically. Assume $x \succsim y \succsim z$. Then y belongs to the shaded area in the graph. For any such y, the no-worse-then-y-set is a subset of the no-worse-than-z-set (or its boundary is either the same or above). Thus, since x belongs to the no-worse-then-y-set, it also belongs to the no-worse-than-z-set: thus x is at least as desirable as z, proving transitivity.

Alternative answer (algebraically). Assume $x \succsim y \succsim z$. Then $x_1^2 + x_2^2 \geq y_1^2 + y_2^2 \geq z_1^2 + z_2^2$. Thus, $x_1^2 + x_2^2 \geq z_1^2 + z_2^2$. By definition this means that $x \succsim z$, proving transitivity.

H. [max 3 points]

[1 point for correctly stating the property; 2 points for correctly checking the property]

Graphically. Convexity holds if the no-worse-than-z-set is convex for each z. This is not the case as shown in the picture.

Alternative answer (algebraically). The bundles $\left(\frac{3}{2}, 0\right)$ and $\left(0, \frac{3}{2}\right)$ are both at least as desirable as $z = (1, 1)$. Their linear combination $\frac{1}{2}\left(\frac{3}{2}, 0\right) + \frac{1}{2}\left(0, \frac{3}{2}\right) = \left(\frac{3}{4}, \frac{3}{4}\right)$ is not, since $\left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2 < (1)^2 + (1)^2$.

I. [max 4 points]

[2 points for correctly stating the property; 2 points for correctly checking the property, only 1 if incomplete/discursive argument]

Graphically, moving to the north-east region from z, leads to a bundle that belongs to the no-worse-than-z-set, but not to the no-better-than-z-set. Thus it is strictly preferred and greed is satisfied.

Alternative answer (algebraically). Greed is satisfied if for each z and each $\varepsilon \equiv (\varepsilon_1, \varepsilon_2) > 0$, $z + \varepsilon \succ z$. By definition, this is equivalent to $(z_1 + \varepsilon_1)^2 + (z_2 + \varepsilon_2)^2 > (z_1)^2 + (z_2)^2$. As this is true for each $\varepsilon_1, \varepsilon_2 \geq 0$ with one of the two different from 0 (i.e. $\varepsilon > 0$), greed holds. One could also solve for the inequality and cancel out some terms...

3 UNCERTAINTY [max 10 points]

J. [max 5 points]

[2 points for correctly defining the risk premium, in words and/or formula]

Let the certainty equivalent of a lottery is the scalar ξ that satisfies $U(\xi, \xi) = U(x_1, x_2)$.

The risk premium is $\mathcal{E}x - \xi$ (or $E[x] - \xi$ or $\frac{1}{2}x_1 + \frac{1}{2}x_2 - \xi$).

[1 point for finding the certainty equivalent]

The certainty equivalent at $(x_1, x_2) = (0, 2)$ satisfies $\frac{1}{2}\xi^2 + \frac{1}{2}\xi^2 = \frac{1}{2}0^2 + \frac{1}{2}2^2$. Thus, $\xi^2 = 2$ and $\xi = \sqrt{2}$.

[2 points for finding the risk premium; only 1 if computational mistakes]

The risk premium is $\frac{1}{2}0 + \frac{1}{2}2 - \xi$. Which is $1 - \sqrt{2}$ (or, approximatively, -0.414).

K. [max 5 points]

[1 point for correctly writing the formula of the index; 1 point for its correct interpretation]

The index of relative risk aversion measures the willingness of an individual to take up a gamble. It is a local index of risk aversion as it is computed at a specific bundle.

It is given by the following formula:

$$\rho(c) = -\frac{cu''(c)}{u'(c)},$$

where $u(c)$ is the cardinal utility (or felicity) function that depends on the scalar c (the payoff at a specific state of nature). (Students might use different notation than c; also x is ok if they do not get confused with the bundle x above).

[2 points for correctly finding the index for the specific case]

In this case $u(c) = c^2$. Thus:

$$\rho(c) = -\frac{cu''(c)}{u'(c)} = -\frac{c \cdot 2}{2c} = -1.$$

[1 point for correctly interpreting]

A relative risk aversion of -1 means that the individual is not risk averse: the individual is risk loving. (And has constant relative risk aversion).

Grading scheme ECON3200 November 2015, Game Theory part

A) A NE is a set of strategies (one for each player) where each player prefers his chosen strategy over all other, given the strategies of the other players.

4 point total: They can write in words or formally. Important to get full score is to indicate that a player makes the choice given the choices of the others.

B) DD is unique NE.

4 points total: they need to explain (briefly) what they have done not just draw lines. E.g., Defect is BR for each player no matter what the other does.

C) An efficient strategy (and here strategy= actions for both players) is one where there exists no other strategy for which both are (weakly) better off.

5 points

D) DC, CD and CC are efficient.

7 points: I expect many to say that only CC is efficient not noticing/understanding that CD and DC cannot be improved upon for both players. Hence saying CC should give 1+1 (with motivation) and an additional 3+2 (with motivation) for saying also DC.

E)

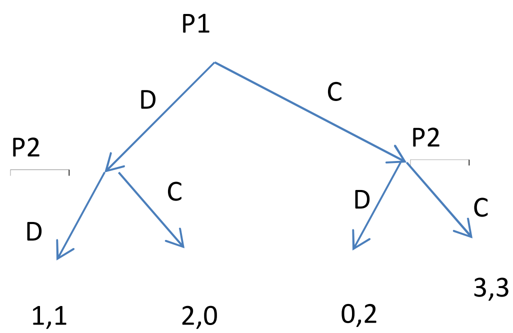
P.1↓ P.2→	Defect	Cooperate
Defect	1,1	2,0
Cooperate	0,2	3,3

5 points: I expect some will put -2 instead of 2. This has no consequence for the further analysis. So such students should get only 2 points here (unless they clearly explain their interpretation in which case they should get no deduction). Otherwise a correct matrix gives 4 points and some kind of motivation 1 point.

F) DD and CC and a mixed where both players play each strategy with prob 0.5.

5 points: They need to explain (briefly) what they have done not just draw lines. Deduct 1 point for missing the mixed strategy.

G)



5 points: some may think that the DC alternative should yield 4,0 payoff (thinking that if P1 deviates first and P2 plays C, then P1 should not be punished). This should lead to a deduction of 2 points (unless they explain how they interpret in which case they should get no deduction), it has no consequence for the further analysis.

H) The unique SPNE is P1 plays C; P2 plays D after D and C after C. CC is outcome.

5 points: Only backward induction without writing out equilibrium yields 2 points. If they don't write out the full equilibrium strategy of P2, then they should get 3 points (this has been emphasized in class).

I) The first game is classic PD, the second is a (pareto) coordination game, the third is a sequential coord game. The first obviously yields suboptimal outcomes. In the second a punishment (deviation from the norm) which implies a cost still improves welfare since in eq the punishment never happens. Still in second game there is a risk of miscoordination. By playing sequentially the coord problem is resolved in game three.

10 points for a discussion like this. 2 points for only saying difference in outcome without highlighting what it is that resolves the problems.