Exam solutions

Question 2 [20 points in total]

Technology: $f(x_1, x_2, x_3) = \min\{x_1, x_2 + x_3\}$.

a) determine the cost function [Up to 12 points]

First, assume $w_1, w_2, w_3 > 0$.

Method 1, solve the standard KKT conditions.

Method 2 [better; I explained student that, in these special cases, they can argue for the solution, graphically or by explaining in words/math their reasoning]. Without solving the Lagrangean, note that $x_1^* \neq x_2^* + x_3^*$ cannot be cost minimizing for any production level. Assume that $x_1^* < x_2^* + x_3^*$, then it is possible to reduce the quantity of inputs 2 or 3, while keeping input 1 unchanged, i.e., $x_1^* = x_1' < x_2' + x_3' < x_2^* + x_3^*$ and still produce the same output $f(x_1^*, x_2^*, x_3^*) = f(x_1', x_2', x_3') = x_1^*$. The conclusion follows from positivity of prices. Thus, at an optimum, $x_1^* = x_2^* + x_3^*$.

Now, x_2 and x_3 are perfect substitutable. It is immediate to show that $x_2^* = 0$ if $w_2 > w_3$ and $x_3^* = 0$ if $w_2 < w_3$.

Thus, the conditional inputs demands are:

$$x_1^* = y$$

$$x_2^* = \begin{cases} y & \text{if } w_2 < w_3 \\ k & \text{if } w_2 = w_3 \\ 0 & \text{otherwise} \end{cases}$$

$$x_3^* = \begin{cases} 0 & \text{if } w_2 < w_3 \\ y - k & \text{if } w_2 = w_3 \\ y & \text{otherwise} \end{cases}$$

The cost function is given by $C = \sum w_i x_i^* = w_1 y + \begin{cases} w_1 y & \text{if } w_2 < w_3 \\ w_1 k + w_2 (y - k) & \text{if } w_2 = w_3 \\ w_2 y & \text{otherwise} \end{cases}$.

Substituting gives: $C = (w_1 + \min\{w_2, w_3\})\, y$

b) [Up to 4 points]

When prices are $w_1 = 1$, $w_2 = 2$, and $w_3 = 3$, the demands of inputs are:

$$x_1^* = x_2^* = y$$

$$x_3^* = 0$$

c) [Up to 4 points]

Inputs 2 and 3 are perfectly substitutable: there is a level of prices for which there are infinite combinations of inputs which are cost minimizing; otherwise, only the cheapest input is demanded. Input 1 and the composite input $x_2 + x_3$

are perfect complements: the conditional demands of input 1 is independent of the prices of the other inputs; the conditional demand of the composite input $x_2 + x_3$ is independent of the price of input 1.

Question 3 [15 points in total]

Utility function: $u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$.

a) determine the Walrasian demand functions [Up to 10 points]

- Write the Lagrangean (better if they take the log transformation of the utility function)

$\mathcal{L} = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} + \lambda (m - p_1 x_1 - p_2 x_2)$

- Argue that the budget constraint holds with equality (by monotonicity of preferences or increasing utility function)
- Argue that there are no corner solutions (infinite marginal utility as a good approaches 0)
- Write the FOCs (they may omit the * as I will do, but better if they state so):

$$\begin{cases} \frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} = \lambda p_1 \\ \frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}} = \lambda p_2 \\ m = p_1 x_1 + p_2 x_2 \end{cases}$$

- Combine the first 2 FOCs to obtain:

$$\frac{\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}}{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}} = \frac{p_1}{p_2} = \frac{1}{2} \frac{x_2}{x_1}$$

Thus:

$$x_2 = 2 \frac{p_1}{p_2} x_1$$

- Substitute in the budget constraint and solve for the demand of x_1:

$$m = p_1 x_1 + p_2 2 \frac{p_1}{p_2} x_1 = 3 p_1 x_1$$

$$x_1^* = \frac{1}{3} \frac{m}{p_1}$$

- Substitute in the above:

$$x_2^* = 2 \frac{p_1}{p_2} \frac{1}{3} \frac{m}{p_1} = \frac{2}{3} \frac{m}{p_2}$$

- The proportion of money spent on good 1 is $\frac{m}{p_1 x_1^*} = \frac{1}{3}$
- the proportion of money spent on good 2 is $\frac{m}{p_2 x_2^*} = \frac{2}{3}$

b) [up to 5 points]

- Substitute the demands in the utility function to get:

$$v(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} = \left(\frac{1}{3}\frac{m}{p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3}\frac{m}{p_2}\right)^{\frac{2}{3}} = \left(\frac{1}{3p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3p_2}\right)^{\frac{2}{3}} m$$

- Now, to verify Roy's identity, first compute the derivatives of v with respect to m and p_1:

$$v_m = \left(\frac{1}{3p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3p_2}\right)^{\frac{2}{3}}$$

$$v_{p_1} = \frac{1}{3} p_1^{-2} \frac{1}{3} \left(\frac{1}{3p_1}\right)^{-1} \left(\frac{1}{3p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3p_2}\right)^{\frac{2}{3}} m$$

- Roy's identity tells that

$$x_1^* = -\frac{v_{p_1}}{v_m} = \frac{\frac{1}{3} p_1^{-2} \frac{1}{3} \left(\frac{1}{3p_1}\right)^{-1} \left(\frac{1}{3p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3p_2}\right)^{\frac{2}{3}} m}{\left(\frac{1}{3p_1}\right)^{\frac{1}{3}} \left(\frac{2}{3p_2}\right)^{\frac{2}{3}}} = \frac{1}{3}\frac{m}{p_1}$$

so it is verified.

Geir B. Asheim, 2018.11.01

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Question 6

One Nash equilibrium (S,S) Pareto-dominates the other (H,H), but (H,H) is more safe if you are uncertain about the behavior of your opponent. Both equilibria may occur.

Question 7

For each opponent strategy, the preference of a player remains the same. Therefore, the set of Nash-equilibria is the same. However, if a and b exceed 1, then (H,H) becomes Pareto-dominant, while remaining more safe.

Question 9

(H,H) Pareto dominates (S,S) and is more safe. It is natural to predict that the player will coordinate on (H,H). The subtraction of a=4 and b=4 has changed which equilibrium is Pareto-dominant.

Question 11

(a) Even though the striker is more skilled at kicking at the goalie's left, he scores with probability 1 – ab/(a+b) if he kicks to the goalie's left and the same, 1 – ab/(a+b), if he kicks to the goalie's right. It is not wise to use this comparison for judging the striker's relative scoring ability. (Weight: 5/10)
(b) The probability of scoring if both choose L is 1 – a, and the probability of scoring if both choose R is 1 – b. Since 1 – a > 1 – b, this shows correctly that the kicker is more skilled when choosing L. (Weight: 5/10)

Question 12

(a) Players would expect that the other would leave the money for a "long" time, making it profitable to do also. This effect is stronger if N is large. (Weight: 2/10)
(b) Backward induction implies that the last player will take, the penultimate player will take and so on, leading each player to use the strategy where take is chosen at every decision node. (Weight: 6/10)
(c) This means that backward induction might not always predict actual behavior. (Weight: 2/10)