The exam consists of 8 problems. They count as indicated. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are problems or parts of problems that you do not have time to solve.

Part 1. Microeconomics.

Problem 1 (6 %)

Andrea's preferences over cars satisfy the following: a car is at least as desirable as another one if and only if it is at least as fast. Do Andrea's preferences over cars satisfy completeness, transitivity, reflexivity? Briefly justify your answers.

Andrea's preferences satisfy the properties. For any two cars, Andrea either prefers the faster one (if one is faster) or is indifferent (if they are equally fast), so these can be ranked. For any three cars such that Andrea weakly prefers the first to the second and the second to the third, it must be true that the first one is at least as fast as the third one, so that Andrea weakly prefers it and transitivity holds. Since any car is at least as fast as the car itself, Andrea fids any car at least as desirable as the car itself: reflexivity holds.

Problem 2 (9 %)

Barbara's preferences are represented by the utility function $u(x_1, x_2) = x_1 + x_2^2$. Is the first order condition of the utility maximization problem informative? Why? What is the share of her income that she would optimally spend on good 1, if income is $m = 100$ and prices are $p_1 = p_2 = 10$?

Barbara's preferences are not concave. So the FOC of the UMP does not give a maximum, but a minimum. Thus, there will be corner solutions: either $x_1 = 10$ *or* $x_2 = 10$. *The second case leads to a higher utility, so the share of income to spend on good 1 is 0.*

Problem 3 (8 %)

Ciro considers coffee and sugar as perfect complements with a fixed proportion (1 spoon of sugar for each cup of coffee). What do we know about the substitution and income effect resulting from a reduction in the price of sugar? Graphically sketch the Marshallian and Hicksian demand functions of Ciro.

With perfect complements the substitution effect is 0 and the Hicksian demand is vertical (compensated changes in prices do not affect demand). All the change in demand comes from the income effect, which is positive. So, the Marshallian demand is negatively sloped.

Problem 4 (15 %)

Del Ta is a company that produces yoghurt y. The production involves two inputs, milk m and bacteria b. The production function is $f(m, b) = m^{\frac{1}{3}} b^{\frac{1}{3}}$. The firm is price-taker. Determine the cost and profit functions.

Cost function. First notice that f is continuous and increasing, so (for positive input prices) it is never optimal to produce more than y. Second, for any y>0, m and b need to be positive, so the solution is interior. Set the Lagrangean of the cost minimization problem:

$$L = p_m m + p_b b + \lambda\left(y - m^{\frac{1}{3}} b^{\frac{1}{3}}\right).$$

The FOCs are (I omit the asterisks for notational convenience):

$$p_m = \frac{1}{3}\lambda m^{\frac{1}{3}-1} b^{\frac{1}{3}}$$

$$p_b = \frac{1}{3}\lambda m^{\frac{1}{3}} b^{\frac{1}{3}-1}$$

$$y = m^{\frac{1}{3}} b^{\frac{1}{3}}$$

Solving from the first two gives $b^* = \frac{p_m}{p_b} m^*$ *. Substituting in the latter, gives:* $m^* = \left(\frac{p_b}{p_m}\right)^{\frac{1}{2}} y^{\frac{3}{2}}$*.* *Substituting again gives:* $b^* = \left(\frac{p_m}{p_b}\right)^{\frac{1}{2}} y^{\frac{3}{2}}$*. And determines a formula for the cost function of:*

$$C = p_m m^* + p_b b^* = 2 p_m^{\frac{1}{2}} p_b^{\frac{1}{2}} y^{\frac{3}{2}}$$

Profit function. The output optimization problem is: $\max py - 2p_m^{\frac{1}{2}} p_b^{\frac{1}{2}} y^{\frac{3}{2}}$*. Since the cost function is convex, the optimization problem is well-defined and the second order condition for a maximum will hold. The FOC is (I omit the asterisks for notational convenience):*

$$p = 3 p_m^{\frac{1}{2}} p_b^{\frac{1}{2}} y^{\frac{3}{2}-1}.$$

Thus, the optimal output is $y^* = \frac{1}{9}\frac{p^2}{p_m p_b}$*. The profit function is:*

$$\Pi = py^* - 2p_m^{\frac{1}{2}} p_b^{\frac{1}{2}} y^{\frac{3}{2}} = \frac{1}{9}\frac{p^3}{p_m p_b} - 2p_m^{\frac{1}{2}} p_b^{\frac{1}{2}}\left(\frac{1}{9}\frac{p^2}{p_m p_b}\right)^{\frac{3}{2}}$$

$$\Pi = \frac{1}{9}\frac{p^3}{p_m p_b} - 2p^3 3^{-3} p_m^{\frac{1-3}{2}} p_b^{\frac{1-3}{2}} = \left(\frac{1}{9} - \frac{2}{27}\right)\frac{p^3}{p_m p_b} = \frac{1}{27}\frac{p^3}{p_m p_b}$$

Problem 5 (12 %)

Emilia's vN-M utility function is $u(c_1, c_2, \pi_1, \pi_2) = \pi_1\sqrt{c_1} + \pi_2\sqrt{c_2}$, where c_1 and c_2 are monetary payoffs in states 1 and 2 and π_1 and π_2 are the probabilities associated to such states. Emilia is offered the possibility to take a gamble that pays either 36 or 64 with equal probability. What is Emilia's reservation price for this gamble? What is her risk premium?
Consider now a gamble that pays 4 times as much in each state (i.e., $4 * 36$ and $4 * 64$). Would Emilia be willing to pay 4 times as much for such a gamble? What index of risk aversion is constant under such behavior?

The certainty equivalent of the gamble is 49. So, she would be willing to pay at most 49 for it. The risk premium is given by the difference between the mean and the certainty equivalent, i.e. 50-49=1.

The certainty equivalent of the second gamble is $4 * 49$*. So, she is willing to pay exactly 4 times as much. The index of relative risk aversion is the one that illustrates how risk behavior changes with proportional changes of the stakes. Here,* $\rho = -\frac{c * u''}{u'} = -\frac{c * .25c^{-1.5}}{-.5c^{-.5}} = 2$ *is constant.*

Part 2. Game Theory.

Problem 6 (24 %)

True or false? For each of the statements, if true, try to explain why it is true, and if false, try to explain why it is not true or provide a counterexample. *Partial credit also for correct true/false answer without explanation (or counterexample).*

(a) The procedure of backward induction identifies a unique strategy profile in any finite extensive form game of perfect information. *False: With payoff ties, there might be more than one strategy profile identified by backward induction. An example should be provided. Partial credit if the statement is claimed to be true, and the process of backward induction is explained correctly.*

(b) In the sequential bargaining game with an infinite number of periods (the Rubinstein model) the bargaining weights are divided equally between the players. *False: The first proposer has bargaining weight $1/(1+\delta) > ½$.*

(c) In an infinitely repeated game, a strategy profile is a subgame perfect Nash equilibrium only if the players in each repetition play according a Nash equilibrium of the stage game. *False: An example should be provided; e.g. prisoners' dilemma with Nash reversion.*

(d) When constructing the Bayesian normal form of a static game of incomplete information we consider the situation from an ex ante perspective, where no player has yet received information of their own type. *True. The way to construct the Bayesian normal form should be sketched.*

(e) A dynamic Bayesian game models screening if the uninformed player moves before the informed player. *True.*

(f) In a perfect Bayesian equilibrium each player's strategy specifies optimal actions given his beliefs and the strategies of the other players, and the beliefs are always consistent with Bayes' rule. *False. Not all beliefs can be constructed on basis of Bayes' rule. A good example would be a signaling game with two types, where the two types pool. Then there is some action after which Bayes' rule cannot be used.*

Problem 7 (10 %)

Consider a normal form game with two players. Define the concepts of *best response* and *strictly dominated* strategy. *A player's pure strategy is a best response to a belief about the strategy played by the opponent if no other pure strategy for the player yields a higher payoff given this belief. The belief about the strategy played by the opponent might put positive probability on more than one opponent strategy. A player's pure strategy is strictly dominated if there is another pure or mixed strategy for the player that yields a higher payoff for every pure strategy of the opponent.* How do they relate to each other? *A pure strategy is strictly dominated if and only if there is no belief about the strategy played by the opponent such that this strategy is a best response.* In particular, can a strategy that is strictly dominated be a best response to any belief about the behavior of the other player? *No.* And is a strategy that is not a best response to any belief about the behavior of the other player strictly dominated? *Yes.* Explain!

Problem 8 (16 %)

(a) Consider the following normal form game, where player 1's pure strategies are U and D, and player 2's strategies are L, M, N and R. Player 1's payoffs are listed first, player 2's second.

	L	M	N	R
U	1, 3	2, 2	3, 0	0, 1
D	0, 0	3, -1	2, 2	1, 1

What strategies are rationalizable? *U for player 1 and L for player 2.* What strategy profiles are a Nash equilibrium? *(U, L).*

(b) Assume now that player 1 first makes her choice of U or D, and that player 2 can observe player 1's choice before making his choice of L, M, N or R. The payoffs of the players as functions of their actions are the same as under part (a). What is the extensive form of this dynamic game? *The game tree must be drawn.* How many strategies does player 2 have in this extensive-form game? $4 \cdot 4 = 16$. What strategy profiles are a subgame perfect Nash equilibrium? *(D, (L,N)).*