D.T. Rowland: Demographic methods and concepts
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1.5 Comparative measures

See also: Chapters 2, 3, 4, 7, 8, Appendix A or "Population Handbook" at www.prb.org/pdf/PopHandbook_Eng.pdf

Demographic rates

Rates are the most widely used comparative measures of population change. Ideally, demographic rates show the relationship between the number of demographic events (in the numerator) and *the population at risk* of experiencing them (in the denominator). The population at risk is the population that could potentially experience a particular event, such as giving birth or migrating, in a specific period of time. The concept of exposure to risk (Cox 1970: 22–23) is important in demography, since measures based on the population at risk are more accurate and more informative than those based simply on the total population. For instance the population at risk of having a baby in the year 2000 comprised only females of reproductive age at that time. Being 'at risk' does not imply that a particular event is necessarily unwelcome; rather it reflects wider usage of an expression that originated in mortality studies, where there is special interest in the risk of dying (Pressat 1985: 74).

Poisson rate

Exposure time

Often, the most convenient approximation to the population at risk is the total population at mid-year. Mid-year is assumed to be the point by which half the changes have occurred. It could be misleading to relate changes to the start-of-year or end-of-year populations if substantial growth or decline took place. If the population on 1st January was 10 000 and on 31st December 40 000, each total would produce a markedly different rate.

When unavailable in published statistics, the mid-year population may be calculated as the mean, or average of the population at the start and end of the year:

$$(P_0 + P_n)/2 \quad \text{e.g. } (10000 + 40000)/2 = 25000$$

Alternatively, the mid-year population can be calculated as the initial population plus half the numerical growth during the period:

$$P_0 + \frac{1}{2}(P_n - P_0) \quad \text{e.g. } 10000 + 0.5(40000 - 10000) = 25000$$

This leads to the more general expression for the population at any point in a year, i.e. the initial population plus the required fraction of the annual increase:

$$P_0 + \frac{1}{n}(P_n - P_0)$$

where n is the number of periods into which the year is divided.

For example, to obtain the population on the 31st January use either:

$$P_0 + \frac{1}{12}(P_n - P_0)$$

which is based on the average monthly change, or:

$$P_0 + \frac{31}{365}(P_n - P_0)$$

which is based on the average daily change.

Rates are commonly multiplied by 100 or 1000 to produce figures greater than 1, since whole numbers are more easily comprehended than decimal fractions. Some rates for rare events, such as deaths from certain diseases, are specified per 100 000 or even per million to reach whole numbers.

Other types of measures

When the population at risk is unavailable, the derived figure is strictly a ratio rather than a rate. A ratio expresses the size of a number relative to another convenient number. One of the best-known in demography is the *sex ratio*, the number of males per hundred females (males/females × 100). Although a simple measure, it is relevant to understanding matters as diverse as marriage opportunities, the representation of women in the labour force and, at later ages, the consequences for family support of sex differences in life expectancy. Denominators for ratios are selected according to the availability of data and ease of understanding.

Rates and ratios are essential bases for many comparative studies, but equally common are proportions and percentages. A proportion is a ratio in which the denominator includes the numerator. For example, the proportion of older people in the population of France at the 1990 Census was 0.15 – obtained by dividing the numbers aged 65 and over in France (8 347 869) by the total population of France (56 634 299). Since proportions are necessarily decimal fractions, such ratios are often multiplied by 100 to produce percentages, which are easier to read; thus 15 per cent of the population of France in 1990 were in the older ages.

A further major type of comparative measure is the probability, which ranges between 0 and 1. There are statistical arguments about the correct definition of a probability, but for practical purposes a probability is the ratio of events in a fixed period of time to the initial population at risk. Probabilities in demographic work are always based on the initial population. The probability of dying at age 100, for example, is based on the number of people who celebrated their 100th birthdays (the initial population). If 2500 women reached 100 and 700 of them died before turning 101, their probability of dying at age 100 would be 700/2500 = 0.28. This is a fairly typical figure for centenarian females in more developed countries.

Table 1.2 summarizes the characteristics of the comparative measures discussed in this section. It is often preferable for absolute numbers to accompany such figures to aid comparisons and assist readers to gauge the magnitude of changes. While the table lists the most common types of comparative measures it is not exhaustive. Others include index numbers, rate ratios and averages.

Table 1.2 Types of comparative measures in demography

Rate	The ratio of the number of demographic events (e.g. births) to the population at risk of experiencing the event.
Ratio	The size of a number relative to another convenient number.
Proportion	A ratio in which the denominator includes the numerator.
Percentage	A proportion multiplied by 100.
Probability	The ratio of the number of demographic events to the initial population at risk of experiencing them. Closed population

1.6 Basic measures of change

Crude birth and death rates

The above summary of comparative measures provides the context for recognizing the advantages and disadvantages of the main measures of population change used in cross-national studies. This section discusses the basic measures of population change, as employed for examining trends in population growth and its components.

The crude birth rate (CBR) and the crude death rate (CDR) are based on the total population. They are described as 'crude' because they are unrefined in relation to the population at risk. The crude birth and death rates are key descriptors of the demographic transition (Figure 1.2 and Box 1.2), enabling countries to be compared, irrespective of their total populations:

$$\text{Crude birth rate} = \frac{\text{number of live births in a year}}{\text{mid-year population}} \times 1000$$

$$\text{Crude death rate} = \frac{\text{number of live deaths in a year}}{\text{mid-year population}} \times 1000$$

However, there are several characteristics of these rates that affect their calculation and interpretation:

- The rates are 'crude' because they make no reference to smaller groups that might better represent the population likely to experience the event. In more developed countries, for example, the majority of deaths are in the older ages.
- Crude rates are calculated for calendar years. If the birth or death data do not refer to a calendar year the average number of events per year is used instead (Barclay 1966: 35–36). Thus the numerator refers either to the number of events in a calendar year – such as births or deaths – or to the average number per year. Similarly, the denominator refers either to the mid-year population, or to the average population for the period.

Thus if the total number of births for the period 1995–2000 was 25 000, the population in 1995 was 300 000 and 350 000 five years later, the average crude birth rate for the period would be: (25 000/5)/((300 000 + 350 000)/2) × 1000 = (5000/325 000) × 1000 = 15.4 per thousand. Averaging the number of events gives satisfactory results, but it is preferable to work with annual data if they are available. Use of the mean population assumes that the population changes by a constant number each year, which is commonly an acceptable assumption for intervals of five years or less.

- Despite their use for cross-national comparisons and comparisons through time, the age structure of populations can have a substantial effect on crude rates. For example, a developing country population with a high proportion of young people might be expected to have a relatively low crude death rate. Accordingly, standardization techniques (see Chapter 4) are employed to refine comparisons by removing the influence of age structure differences, as well as other compositional influences.

Since statistics for calculating crude rates are most widely available, the occurrence of other phenomena is sometimes expressed in the same way as a convenient first approximation. Examples are the crude marriage rate and the crude divorce rate (see Chapter 7). Both are based on the total population, taking no account of the numbers of single people of marriageable age, or of the numbers married and, therefore, potentially at risk of separation or divorce.

Rates of natural increase and net migration

From the crude birth and death rates is derived the *(crude) rate of natural increase* (RNI), which is simply the difference between them:

$$\text{RNI} = \text{CBR} - \text{CDR}$$

Alternatively, the rate of natural increase can be obtained as:

$$\text{Rate of natural increase} = \frac{\text{births} - \text{deaths in a year}}{\text{mid-year population}} \times k$$

where $k = 1000$ or 100.

In the demographic histories of most of the now more developed countries, RNI never exceeded 1.5 per cent – the United States was an exception (McNicoll 1984: 181). Less developed countries have had peaks of double this figure, and rates of natural increase above 3 per cent are still common in Africa.

Table 1.3 presents examples of the calculation of basic rates for Japan in the 1950s and 1990s. Although RNI is equal to CBR – CDR in Table 1.3, the result may be divided by 10 to express RNI as a rate per cent. Japan's rate of natural increase was close to zero per cent in the 1990s.

Table 6.5 Basic measures of mortality

Measures	Examples
Crude death rate (CDR)	Egypt 1990 (estimates)
Crude death rate $= \frac{\text{number of deaths in a year}}{\text{mid-year population}} \times 1000$	Deaths 534 000
or	Mid-year pop. 52 426 000
$CDR = \frac{D}{P} \times 1000$	CDR 10.2

Age-specific death rate (ASDR)

$$\text{Age-specific death rate} = \frac{\text{deaths in calendar year at age } x}{\text{mid-year population aged } x} \times 1000$$

or

$$ASDR = \frac{D_x}{P_x} \times 1000$$

$0 \leq ASDR \leq 2$

Probability of dying (q), one year

$$q = \frac{ASDR}{1 + \frac{1}{2} ASDR}$$

Probability of dying ($_nq$) over a period of n years

$$_nq = \frac{n \cdot ASDR}{1 + \frac{n}{2} ASDR}$$

$0 \leq q \leq 1$

Russian Federation 1995

	Mid-Year Pop. Males	Deaths	(‰) ASDRs	(‰) q_5
45–49	4 499 834	86 647	19.3	92.1
50–54	2 667 863	72 457	27.2	127.3
55–59	4 296 680	145 374	33.8	155.8
60–64	2 909 878	133 360	45.8	205.5
65–69	2 859 230	170 747	59.7	254.7
70–74	1 209 717	91 776	75.9	319.0
75–79	609 267	64 710	106.2	418.6

Table 7.1 Basic measures of fertility

Measures / **Examples**

Crude birth rate (CBR)

$$CBR = \frac{\text{number of live births in a year}}{\text{mid-year population}} \times 1000$$

or

$$CBR = \frac{B}{P} \times 1000$$

Egypt 1990 (estimates)
Births 1 737 000
Mid-year pop. 52 426 000
CBR = 33.1

General fertility rate (GFR)

$$GFR = \frac{\text{live births in a year}}{\text{mid-year pop. of females aged } 15-49} \times 1000$$

or

$$GFR = \frac{B}{F_{15-49}} \times 1000$$

Egypt 1990 (estimates)
Births 1 737 000
Women 15–49 12 423 000
GFR = 140

Child–woman ratio (CWR)

$$CWR = \frac{\text{number of children aged 0–4}}{\text{number of females aged 15–49}} \times 1000$$

or

$$CWR = \frac{P_{0-4}}{F_{15-49}} \times 1000$$

Egypt 1990 (estimates)
Children 0–4 7 588 000
Women 15–49 12 423 000
CWR = 611

Age-specific fertility rate (ASFR)

$$ASFR = \frac{\text{no. of births in a year to women aged } x \text{ to } x+n}{\text{mid-year pop. of women in aged } x \text{ to } x+n} \times 1000$$

or, for single years of age

$$ASFR = \frac{B_x}{F_x} \times 1000$$

where x is a single year of age, e.g. age 20 years
For grouped ages

$$ASFR = \frac{B_i}{F_i} \times 1000$$

where i is a five year age group, e.g. 20–24 years

Russian Federation 1995

Age	Mid-year pop. females	Births	ASFRs
15–19	5 335 672	238 019	44.6
20–24	4 986 816	561 796	112.7
25–29	4 645 374	309 371	66.6
30–34	5 802 104	171 115	29.5
35–39	6 448 795	68 219	10.6
40–44	6 013 599	13 073	2.2
45–49	4 845 679	578	0.1
		TFR =	1.3

Sources: Shryock and and Siegel (1973); United Nations (1993 & 2000b); US Census Bureau 2001, *Statistical Abstract of the United States 2000.*

Total fertility rate (TFR)
= sum, for all ages, of ASFR /1000 = $\sum_{x=15}^{x=49} B_x / F_x$
number of children per women

TFR = 5 × (sum of ASFR/1000) when age-intervals of five years

The total fertility rate in 2002 in Israel was 2.9 births per woman (or 2,900 births per 1,000 women). That is, if 2002 age-specific rates continued unchanged, women in Israel would average 2.9 children each during their childbearing years.

In some developing countries, the TFR is more than five children per woman. In most developed countries, it is below two.

Gross Reproduction Rate

GRR = TFR * proportion girls

The gross reproduction rate (GRR) is the average number of daughters that would be born to a woman (or group of women) during her lifetime if she passed through her childbearing years conforming to the age-specific fertility rates of a given year. This rate is like the TFR except that it counts only daughters and literally measures "reproduction"—a woman reproducing herself by having a daughter.

Net Reproduction Rate

The net reproduction rate (NRR) is the average number of daughters that would be born to a woman (or group of women) if she passed through her lifetime from birth conforming to the age-specific fertility and mortality rates of a given year. This rate is like the GRR, but it is always lower because it takes into account the fact that some women will die before completing their childbearing years.

Country	1993 GRR	1993 NRR
Burkina Faso	3.50	2.41
United Kingdom	0.86	0.85

In 1993, Burkina Faso had a GRR of 3.50, while that of the United Kingdom was only 0.86. That means that, if 1993 fertility levels were to continue, a woman in Burkina Faso would produce 3.5 daughters, on average, during her lifetime. In the United Kingdom, by contrast, a woman would produce less than one daughter on average during her lifetime.

In Burkina Faso, one daughter would die, on average, before completing her childbearing years.

of children a woman would have if the fertility rates for a given year applied to her throughout her reproductive life. (See box below showing how the TFR is calculated.)

The TFR is a *synthetic* measure; no individual woman is very likely to pass through three decades conforming to the age-specific fertility rates of any single year. In reality, age-specific rates change and fluctuate from year to year, even if only gradually. For example, women who were ages 15-19 in 2003 may delay childbearing longer than women ages 15-19 in, say, 1990. They would lower the TFR a bit in 2003 but then raise it several years later when they begin their childbearing. Thus, year-to-year fluctuations in the TFR may reflect changes in the *timing* of births rather than changes in the average number of children women bear. The TFR is one of the most useful indicators of fertility because it gives the best picture of how many children women are currently having.

Calculating the Total Fertility Rate

Israel's TFR, 1994

Age of women	(1) Number of women	(2) Number of births [illegible]	(3) Birth rate [illegible]	(4) Age-specific birth rates [illegible]
15-19	244,000	4,474	.018	.090
20-24	225,800	28,013	.124	.620
25-29	194,200	36,440	.188	.940
30-34	182,300	27,402	.150	.750
35-39	181,400	14,044	.077	.385
40-44	177,600	3,176	.018	.090
45-49	151,100	182	.001	.005
			Sum	[illegible]

The rates in column (3) simulate the likelihood of a woman giving birth during each year of her childbearing years—that is, they approximate the "risk" of having a birth. Multiplying each of these rates by five provides the number of children she would have for each five-year period. Each woman is subject to the annual "risk" of a birth five times in each age group; for example, 0.124 when she is 20, 0.124 when she is 21, and so on. Summing the rates for all age categories results in the number of children she would have by age 49—the total fertility rate.

In practice, a maternal death is defined as the death of a woman while pregnant or within 42 days of termination of pregnancy from any cause related to or aggravated by the pregnancy or its management but not from accidental or incidental causes.

Life Expectancy

Life expectancy is an estimate of the *average* number of additional years a person could expect to live if the age-specific death rates for a given year prevailed for the rest of his or her life. Life expectancy is a hypothetical measure because it is based on current death rates and actual death rates change over the course of a person's lifetime. Each person's life expectancy changes as he or she grows older and as mortality trends change.

> If the age-specific death rates for 2000 remain unchanged, males in brazil born in 2000 can expect to live 65 years on average. females can expect to live 73 years.

Because life expectancy differs significantly depending on sex, present age, and race, these categories are usually given separately. Life expectancy at birth is the most commonly cited life expectancy measure. It is a good indicator of current health conditions.

Life expectancies differ widely among countries. In 1996, life expectancy was 46 in Malawi compared with 80 in Japan. Women in Japan enjoy the world's highest life expectancy, 83 years in 1996.

It should be noted that low life expectancies in developing countries are in large part the result of high infant mortality rates. In 1994, for example, life expectancy at birth for females in Bangladesh was 58 years, but if a Bangladeshi female survived to age 1, she could expect to live to age 62.

The Life Table

The life table, one of the most powerful tools in demography, is used to simulate the lifetime mortality experience of a population. It does so by taking that population's age-specific death rates and applying them to a *hypothetical* population of 100,000 people born at the same time. For each year on the life table, death inevitably thins the hypothetical population's ranks until, in the bottom row of statistics, even the oldest people die.

The box on page 30 contains selected portions of an abridged life table for men in Malaysia in 1995. This table is based on death rates and is abridged to display data at five-year age intervals rather than single years.

Column 1 shows the proportion of each age group dying in each age interval. These data are based on the observed mortality experience of a population. Column 2 shows the number of people alive at the beginning of each age interval, starting with 100,000 at

How Life Tables Work

Abridged Life Table for Males in Malaysia, 1995

	1	2	3	4	5	6
				Persons living		
Age x	Proportion dying in the age interval $_nq_x$	Number living at beginning of age interval l_x	Number dying during the age interval $_nd_x$	in the age interval $_nL_x$	in this and all subsequent T_x intervals	e_x Years of life remaining (life expectancy)
0-<1	.01190	100,000	1,190	98,901	6,938,406	69.38
1-5	.00341	98,810	337	394,437	6,839,505	69.22
5-10	.00237	98,473	233	491,782	6,445,067	65.45
10-15	.00270	98,240	265	490,536	5,953,285	60.60
	.....	......	.....	......	....	.....
65-70	.16050	70,833	11,368	325,743	928,004	13.10
70-75	.25762	59,464	15,319	259,024	602,260	10.13
75-80	.34357	44,145	15,167	182,808	343,237	7.78
80+	1.00000	28,978	28,978	160,428	160,428	5.54

Source: Department of Statistics, Malaysia, 1997.

birth. Each age group contains the population that survived from the immediately preceding group. Column 3 shows the number who would die within each age interval (Column 1 x Column 2 = Column 3).

Column 4 shows the total number of person-years that would be lived within each age interval. Column 5 shows the total number of years of life to be shared by the population in the age interval and in all subsequent intervals. This measure takes into account the frequency of deaths that will occur in this and all subsequent intervals. As age increases and the population shrinks, the total person-years that the survivors have to live necessarily diminish.

Life expectancy is shown in Column 6. The total person-years lived in a given interval plus subsequent intervals, when divided by the number of persons living at the start of that interval, equals life expectancy—the average number of years remaining for a person at a given age interval (Column 5 ÷ Column 2 = Column 6). For example, dividing the number of person-years associated with Malaysian men who survive to age 70 (602,260) by the number of these men (59,464) shows they have an additional life expectancy of 10.1 years.

With age, life expectancy actually rises—a kind of "bonus" for surviving. The 59,464 Malaysian men who survive to age 70, for example, can expect to live more than 10 additional years, well past their life expectancy at birth of 69 years.