ECON 3710/4710 Demography of developing countries

Computer exercise: Check of Norway's age structure 1801 by using stable population theory

The purpose of this exercise is to check whether the age structure of the population of Norway from the Census of 1801 resembles that of a stable population. The method that we will use is the one described in "Fertility and mortality estimation using model stable age distributions", in particular Section B "Evaluation of age distributions". As a secondary result of the evaluation, we will be able to draw some conclusions about the levels of fertility and mortality in Norway in the years before 1801. Read the article mentioned above before you start the exercise.

The age structure of the population of Norway according to the Census of 1801 was as follows

	men	women
0-9	109526	108927
10-19	80662	80219
20-29	63934	73020
30-39	53773	59994
40-49	47164	54280
50-59	34467	38826
60-69	20709	26535
70-79	10166	14497
80+	2539	4085
sum	422940	460413

Source: Historical Statistics 1994, Table 3.3, Statistics Norway.

The numbers are given in ten-year age groups. The article that you have read based its method on five-year age groups, but the method used here will be the same. Because of the cumulated age shares (see below), we will have fewer check points, but the pattern will not be influenced.

Other historical data sources show us that the Crude Birth Rate for Norway in the years 1735-1801 has been rather constant at 30-32 per thousand, and the Crude Death Rate at 24-26 per thousand. We will assume that Norway's population was stable in 1801, or at least close to stability.

In the Excel file called "StableExc2.XLS" I have evaluated the age structure of men, which I will explain below. The purpose of the exercise is that you evaluate the age structure of women by the same method. Open the Excel file, and try to understand what I have done.

1. In columns A, B, and C I have copied the table given above. A check of the total showed that there is a small error in the published number for women: their published sum (460413) is 30 persons higher than the sum of the published age-specific numbers (460383). There is little we can do with this error – I decided to correct the published sum, so that I start from a total number of 460383 women.
2. In columns E and F you see the cumulated shares C(x) for men and women: the share of the population up to age x; cf. Table 143 in the article.
3. An obvious choice was to start with life tables model *North* (model West gave results for men that agreed less well). Next I had to choose a mortality level within the North family that would give a good fit. The earliest estimate of the life expectancy of Norwegian men that I could find in published statistics was 45 years in the period 1821-1830 (and 48 years for women in those years).

4. Assuming an increase in life expectancy between 1801 and 1821, I started at level 11, e_0=41.8 years. From Coale and Demeny's volume "Regional Model Life Tables and Stable Populations", I copied three populations that correspond with level North mortality level 11; see columns I-N, rows 2-18. The first three columns (J-L) apply to intrinsic growth rates r equal to 5, 10, and 15 per thousand, respectively. The shares (ranging from 0.23 to 0.99) in rows 6-13 are C(x) values for a stable population with a standard mortality life table "North level 11", and an intrinsic growth rate as in columns J-L. For each age, I only entered those C(x) numbers that are immediately higher or lower than the empirical values for 1801. For example, at age 10, the empirical number is 0.258963, see column F. The two numbers immediately over and under are 0.2367 (a stable population with r=10 per thousand) and 0.2644 (r=15 per thousand). For the remaining ages I proceeded in a similar way.
5. What value for the intrinsic growth rate r would a stable population (North, level 11) have had with a C(10) equal to the observed 0.258963? The answer can be found by linear interpolation. When C(10)=0.2367 for r=10, and C(10)=0.2644 for r=15, the unknown r-value can be computed as r=10+5*(0.258963-0.2367)/(0.2644-0.2367), see the formula in cell M6. This gives an interpolated r-value of 14.0 per thousand (rounded). This way I also obtained interpolated r-values for ages 20, 30, …, 80; see column M.
6. A stable population with North mortality at level 11 has a Crude Birth Rate (b) equal to 27.95 births per thousand when r equal 5 per thousand; see row 14. For r equal to 10 and 15 per thousand, b is 32.28 and 36.92, respectively. Similar to the intrinsic growth rate, I used linear interpolation to find *that* b-value that would have resulted in the observed C(x)-values, given mortality North, level 11. These interpolated b-values are in column N. The interpolation method to compute them is described in the article, Section 4, Step 2.
7. If Norway's population would have been stable in 1801, with North mortality at level 11, then all interpolated r-values would have been the same, irrespective of age. The same can be said of the interpolated b-values. But I find a decrease in the value of r, from 14 (age 10) to 8 (age 80), and b falls from 36 to 31 per thousand. In other words, the elderly show lower growth than the young. I suspect that level 11 is too low, and thus I check the empirical numbers against level 12, see columns P-U. The slopes of r and b (columns T and U) are less than in the previous calculations, but they are not zero. I repeat my attempts (levels 13, 14, 15, and 16) and find a reasonably flat pattern for level 15, with a life expectancy e_0 equal to 51.4 years; see columns AK-AP.
8. Tables compiled by Coale and Demeny show that such a stable population (North, level 15), with r=16.2 and b=32 per thousand, has a Gross Reproduction Rate of around 2.15 boys per man (remember, this is the male part of the population), and a mean age at death equal to 36 years.
9. Even when I accept level 15, I must be critical: age groups 30-60 have too low values compared to a flat pattern. This can indicate two things: mortality in this age group is higher in reality than level 15 would imply, or the Norwegian population has experienced outmigration in the years before 1801. I find the first explanation credible: a life expectancy of 51.4 years appears to be rather high. The other explanation cannot be excluded either.
   Moreover, when I compute the standard deviation in the interpolated r-values (as a measure of variability), the lowest value results for level 13 (1.01, see row 16. The standard deviation restricted to ages 20-70 is lowest for level 12 (0.71, see row 18). Probably levels 12 and 13 are more appropriate choices.

The assignment now is to repeat the computations for women, given the numbers in column C. Column F, rows 23-30 gives the shares C(x) for women. I have entered C(x) values for a stable population with North mortality at levels 12, 13, 14, 15, 16, and 17; see columns I to BD.

Excel-hint: When you "browse" to the right, it is smart to freeze the left-hand part of the table, so that you can inspect column F with empirical shares all the time. This can be done by putting the marker in cell H1, and next clicking on "View" and "Freeze Panes".