# ECON 3710/4710 Demography of developing countries

## Computer exercise: Compute intrinsic growth rates

The purpose of this exercise is to compute the intrinsic growth rate *r* in a stable population, as implied by a set of age-specific birth rates and age-specific death rates.

We will compute the intrinsic growth rates for two cases: Norway 1987 and Egypt 1997.

Data are given in the Excel file called "StableExc1.XLS". Open this file, and store it in your own directory.

There are three spreadsheets in this file: Norway 1987, Solution Norway 1987, and Egypt 1997.

## 1. Intrinsic growth rate for Norway 1987

Open the sheet called "Norway 1987". It contains information on age-specific fertility and mortality in Norway as of 1987. The data are given in five-year age groups of the mother (column 1).
Column 2 gives the mid points (*y*) of the five-year age groups: 17.5, 22.5, …, 47.5, which we will need in later computations (see Comment 3 on page 6 of the Lecture Notes).
Column 3 contains data on age-specific mortality of these women. These data were extracted from a life table for Norwegian women, more precisely the $L_x$ column ("Exposure time").
Age-specific fertility rates are given in column 4. Since much of stable population theory is restricted to women, the rates apply to female births only.
All the data that are necessary to compute the intrinsic growth rate are given in these columns.

### *1.1 Compute an approximate value of the intrinsic growth rate*

The formula is $r \approx \sqrt[\mu]{NRR} - 1$, where *NRR* is the Net Reproduction Rate, and $\mu$ the mean age at childbearing, computed from age-specific fertility rates.
a. Compute net-fertility rates $\varphi_x$ and the *NRR* in columns 5. The net-fertility rate at each age is the product of the fertility rate and the exposure time. *NRR* is the sum of the net-fertility rates.
b. Compute in column 6 the mean age at childbearing $\mu$ based on the net fertility rates in column 5. This mean age equals $(17.5\varphi_{15\text{-}19}+22.5\varphi_{20\text{-}24}+ \ldots +47.5\,\varphi_{45\text{-}49})/NRR$ .
c. Now you are ready to compute the approximate intrinsic growth rate, using the formula given above. Put the formula in cell G25: later on we will need it as a first approximation when we compute the exact value of *r*.

Excel will compute $\sqrt[\mu]{NRR} - 1$ as follows. First note that this expression can be written as $(NRR)^{1/\mu}$ -1. When Excel raises a number to a power, it uses the "hat" symbol. Thus, assuming that *NRR* is stored in cell E22, and $\mu$ in cell F22, you write in cell G25 the formula "=(E22)^(1/F22)-1".

Why is the value that you obtain for *r* less than zero?

### *1.2 Compute the exact value of the intrinsic growth rate*

We use the trial and error procedure described in the Lecture Notes.
a. Start with an initial guess for *r*, namely the approximate value computed under 1.1.
b. Use column 7 to compute the seven terms (one for each age group) of Lotka's equation, given the initial value of the intrinsic growth rate. Use expression (4) of the Lecture Notes (or rather, in modified form, see Comment 3 on page 6). Again, using the "hat" symbol we write (with $r_1$ in cell G25, and *y* in cell B13), we write "=(1+G25)^(-B13)" to obtain $(1+ r_1)^{-y}$.
c. The sum of terms in column 7 should be not too far from 1, but it will not be exactly equal to 1. We can improve the initial guess $r_1$ by computing $r_2 = \{\text{sum}(r_1)-1\}/\mu$, see "2nd try" in row 26. Once you

have computed $r_2$, you repeat the procedure described under b above. Store the results in column 8. Similarly, compute the 3rd guess $r_3$.
d. Now you see that the new value you have computed for the intrinsic growth rate is not different from the previous one in the sixth decimal. Use column 9 to check whether the sum is close enough to one.

Which value did you find for *r*? How much does it differ from the approximate value computed under 1.1? Comment.

The observed natural growth rate (that is, the difference between observed CBR and CDR) for the Norwegian population in 1987 was positive: +0.022 per cent. Compare with the intrinsic growth rate that you computed, and explain the difference.

e. Given the value of *r* that you found under point d, you can now compute the exact value for the average distance between the two generations: $T = \log(NRR)/\log(1+r)$. Compute *T*, and compare it with the mean age at childbearing $\mu$ that you computed earlier. Comment.

YOU CAN CHECK ALL THE CALCULATIONS FOR NORWAY IN THE SHEET "SOLUTION NORWAY 1987".

## 2. Intrinsic growth rate for Egypt 1997

The sheet "Egypt 1997" has the same set up as the sheet for Norway. Repeat the earlier calculations with data from Egypt. I.e. compute

- an approximate value of the intrinsic growth rate;
- the exact value of the intrinsic growth rate
- the value of the distance between two generations

I found the following results:
*NRR* = 1.527 daughters per woman on average
$\mu$ = 30.0 years
approximate value of $r$ = 0.014217 = 1.4 per cent
exact value of $r$ = 0.014346 (1.4 per cent) after three iterations; sum($r_3$) = 1.000004.
$T$ = 29.74 years.

## 3. "Fewer, later" – China's one-child policy

When it was announced in 1980, China's one-child policy persuaded Chinese couples to have fewer children than earlier generations, and to delay marriage. Use the results that you obtained for Egypt to answer the following questions:
- What is the effect on the intrinsic growth rate (approximate solution) when *NRR* is reduced by 0.45 daughters per woman (which reflects a decrease of the Total Fertility Rate by about one child per woman)? Change the *NRR* by this amount, but keep the same value of the mean age at childbearing.
- What is the effect on the intrinsic growth rate when the mean age at childbearing is increased by one year (leaving the *NRR* unchanged)?