

ECON3120/4120 – Mathematics 2, fall term 06

Problems for Seminar 1, 4/9-8/9-06

1 Consider the function f defined by

$$f(x) = \frac{3-x}{3x-3}$$

- (a) Where is $f(x)$ defined? Compute $f(x)$ when $x = -3$, $x = -1/2$, $x = 1/4$, $x = 3/2$, $x = 3$ and $x = 9$.
- (b) Where is $f(x) \leq 0$? Where is $f(x) \leq 1$?
- (c) Draw the graph of f and see if your answers to (b) are confirmed.
- (d) Define $g(x) = \ln[f(x)]$. Where is $g(x)$ defined? Where is $g(x) > 0$?

2 Use l'Hôpital's rule (or other methods) to find:

(a) $\lim_{x \rightarrow 3} \frac{3x^2 - 27}{x - 3}$ (b) $\lim_{x \rightarrow 0} \frac{e^{-3x} - e^{-2x} + x}{x^2}$ (c) (*) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + \frac{1}{2}x} - x)$

3 (a) The equation $e^L + KL = Ke^K$ defines L as a differentiable function of K . Find an expression for dL/dK .

(b) If $z = F(u, v, w)$ and $u = f(x, y)$, $v = e^{-x}$, and $w = \ln y$, find an expression for $\partial z/\partial x$ and $\partial z/\partial y$.

4 Find the differential of z expressed in terms of the differentials of u and v :

(a) $z = uv^2$ (b) $z = u^2/v^3$ (c) $z = F(u^2, v^3)$ (d) $z = u^2 - f(u + v)$

5 The following system defines u and v as C^1 functions of x and y around the point $P = (x, y, u, v) = (1, 2, 1, 1)$:

$$\begin{aligned}u^2 + v^2 &= xy \\xu^2 + yv^2 &= x + y\end{aligned}$$

Differentiate the system. Then find the values of $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$ and $\partial v/\partial y$ at the point P .