

ECON3120/4120 – Mathematics 2, fall term 06

Problems for seminar 7, 23/10-27/10.

1 (Exam problem 30/5-05)

(a) Calculate the determinant of $\mathbf{A}_t = \begin{pmatrix} 0 & t & 1 \\ 4 & -2 & 8 \\ 1 & 1 & 1 \end{pmatrix}$

(b) Find x , y and z such that

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} - \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix}$$

2 Exam problem 142.

3 Find the general solution of the differential equation

$$\dot{x} + \frac{2}{t}x = e^t$$

Find, in particular, the integral curve passing through $(t, x) = (1, 1)$.

4 Given the matrix

$$\mathbf{A}_t = \begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$$

(a) Calculate $|\mathbf{A}_t|$ and show that \mathbf{A}_t^{-1} exists for all t .

(b) Show that for a certain value of t we have $\mathbf{A}_t^3 = \mathbf{I}_3$, where \mathbf{I}_3 is the identity matrix of order 3.

(c) Find the inverse of \mathbf{A}_1 .

(d) Suppose that \mathbf{A} and \mathbf{B} are invertible $n \times n$ -matrices. Show that if $\mathbf{A}'\mathbf{A} = \mathbf{I}_n$, then $(\mathbf{A}'\mathbf{B}\mathbf{A})^{-1} = \mathbf{A}'\mathbf{B}^{-1}\mathbf{A}$.