University of Oslo
Department of Economics

Obligatorisk oppgavesett nr. 1 i ECON3120/4120 Matematikk 2 / Compulsory term paper no. 1 in ECON3120/4120 Mathematics 2

Oppgavesettet foreligger kun på engelsk, men besvarelsene kan skrives på engelsk eller norsk.
This problem set is only available in English, but your term papers may be written in English or Norwegian.

Handed out: Monday, September 10th, 2007

To be handed in: Tuesday, September 25th, 2007 at 1400 hours.

Hand in at the department office, 12th floor

Other information:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, available in English resp. Norwegian at `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc` or `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_nor.doc`
- It is important that the term paper is submitted by the deadline (see above). Term papers submitted after the deadline **will not be read or marked.**[*)]
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on the 12th floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Problem 1 Consider the function $f(x) = 2x^2 - \ln x - 2$, $x > 0$.

(a) We see that $f(1) = 0$. Show that $f(x)$ has exactly one other zero, and that this point is in the interval $(0, 1)$.

(b) Show that f defined on $[1, \infty)$ has an inverse function g, and find $g'(0)$.

Problem 2 The equation system

$$x \cdot y \cdot z \cdot ue^{-u} \cdot ve^{v} = 0$$
$$1x + 2y + 3z + 4u + 5v = 6$$

defines u and v implicitely as continuously differentiable functions of (x, y, z) around some suitable point P.

(a) Differentiate the system (i.e. find the differentials).

(b) Find a general expression for v'_x.

(c) If P has (x, y, z)-coordinates equal to $(1, 1, 1)$, show that $u(1, 1, 1) = v(1, 1, 1) = 0$.

(d) Find an approximation for u in the point where $x = y = 1$ and $z = 1.1$.

Problem 3

(a) Calculate $\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{\sqrt{x+2} - \sqrt{2}}$

(b) Calculate $\lim_{x \to 0^+} x^{1/1000} (\ln x)^{9999}$.

(b) Calculate $\int x e^{\sqrt{x}} \, dx$

(c) Let $f(x) = \int_0^\infty t^x e^{-t} \, dt$ for $x \geq 0$.
Calculate $f(0)$. Then use integration by parts to show that $f(x + 1) = (x + 1)f(x)$.
(This will prove that $f(n) = n!$ when n is a natural number.)

Problem 4 Find the general solution to the differential equation

$$t\dot{x} = x^3 \ln t$$

Problem 5 For points (b) and (c): You can use that $\frac{d}{dx} x^{2/3} = \frac{2}{3} x^{-1/3}$ even for $x < 0$ (that is: the left and right hand sides are both well-defined, and they represent the same value).

(a) Show that $\int_{-12}^{12} x^{2007} e^{x^{1234567890}} \, dx = 0$.

(b) Find the error in the following two calculations

$$\int_{-1}^{1} x^{-1/3}\,dx \tag{i}$$

$$= \left[\tfrac{3}{2}x^{2/3}\right]_{-1}^{1} = \frac{3}{2} - \frac{3}{2} = 0$$

and

$$\int_{-2}^{2} \frac{dx}{x} \tag{ii}$$

$$= \Big[\ln|x|\Big]_{-2}^{2} = \ln 2 - \ln 2 = 0$$

(c) Find the integrals (i) and (ii), if possible.

(d) What can the method used in point (a) tell you about the values of the integrals (i) and (ii)? (Answer cautiously!)