University of Oslo
Department of Economics

Obligatorisk oppgavesett nr. 2 i ECON3120/4120 Matematikk 2 / Compulsory term paper no. 2 in ECON3120/4120 Mathematics 2

Oppgavesettet foreligger kun på engelsk, men besvarelsene kan skrives på engelsk eller norsk.
This problem set is only available in English, but your term papers may be written in English or Norwegian.

Handed out: Wednesday, October 24th, 2007

To be handed in: Wednesday, November 7th, 2007 at 1400 hours. (Or earlier.)

Hand in at the department office, 12th floor

Other information:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, available in English resp. Norwegian at `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc` or `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_nor.doc`
- It is important that the term paper is submitted by the deadline (see above). Term papers submitted after the deadline **will not be read or marked.**[*)]
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

[*)] If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

There are five problems, which will be given approximately equal weight.

Problems 1 through 4 are taken from the spring 2007 exam, which is attached as-is:

Problem 1 Exam ECON4120 spring 2007, problem 1.

Problem 2 Exam ECON4120 spring 2007, problem 2.

Problem 3 Exam ECON4120 spring 2007, problem 3.

Problem 4 Exam ECON4120 spring 2007, do

(a) problem 4 (a)

(b) from problem 4 (b): only the conditions, do *not* show existence and uniqueness of (x^*, y^*).

(c) problem 4 (c).

Do **NOT** do 4 (d). (At least it won't count in your score.)

Problem 5 Indicate true or false; justify briefly, but only briefly. Three wrongs tolerated, score will be based on your best 8 of the below 11 sub-problems. Throughout Problem 5, all boldface capital letters (e.g. $\mathbf{A}$) denote matrices, all boldface minuscles letters (e.g. $\mathbf{x}$) denote vectors, and everything non-boldface denotes numbers, where n and k are integers > 1.

(a) If $\mathbf{AB}$ has an inverse, then $\mathbf{A}$ has an inverse. *(Hint: When is* $\mathbf{AB}$ *defined?)*

(b) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$, then $(\mathbf{A} + \mathbf{B})' = \mathbf{A}' + \mathbf{B}'$.

(c) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$, then $|(\mathbf{A} + \mathbf{B})'| = |\mathbf{A}'| + |\mathbf{B}'|$. *(Hint: Try* $\mathbf{A} = \mathbf{B}$*.)*

(d) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$, then $(\mathbf{A}+\mathbf{B})^2 = \mathbf{A}^2+2\mathbf{AB}+\mathbf{B}^2$. *(Hint: For (d), (e) and (f), calculate the difference between the LHS and the RHS. Is it always zero?)*

(e) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$ and both symmetric, then $(\mathbf{A} + \mathbf{B})'(\mathbf{A} + \mathbf{B}) = \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2$.

(f) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$ and both symmetric, then $(\mathbf{A} + \mathbf{B})'(\mathbf{A} + \mathbf{B}) = (\mathbf{A} + \mathbf{B})^2$ – even when $(\mathbf{A} + \mathbf{B})$ is singular.

(g) If $\mathbf{A}^k = \mathbf{I}_n$ (for some k), then $|\mathbf{A}| = 1$ or -1, and only 1 is possible if k is an odd number.

(h) If $\mathbf{AB} = \mathbf{I}_n$ and $\mathbf{SAAB} = \mathbf{I}_n$, then we can conclude that $\mathbf{S} = \mathbf{B}$ without making the additional assumption that $\mathbf{A}$ is square. *(Hint: Maybe* $\mathbf{A}$ *will be square automatically?)*

(i) If the price vector $\mathbf{p}$, the initial endowment $\mathbf{a}$, and the post-trade endowment $\mathbf{x}$ are all n-vectors, with all prices $p_i > 0$ and $\mathbf{a} \neq \mathbf{0}$, then the equation $\mathbf{p} \cdot (\mathbf{x} - \mathbf{a}) = 0$ in the unknown $\mathbf{x}$, has $n - 1$ degrees of freedom.

(j) If $\mathbf{B} = (b_{ij})_{n \times n}$, where all $b_{ij} = 1$, then the equation $(\mathbf{B} - n\mathbf{I}_n)\mathbf{x} = \mathbf{0}$ has precisely two solutions, namely the null vector and the vector of only ones.

(k) If $\mathbf{A} = (a_{ij})$ is a diagonal matrix, and the equation $\mathbf{Ax} = \mathbf{b}$ has a solution, then the number of degrees of freedom is equal to the number of zeroes on the main diagonal. *(Hint: If there is a zero on the main diagonal, what can you do about that line/column?)*

ECON3120/4120 Mathematics 2

Monday 4 June 2007, 09.00–12.00.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given: A (best), B, C, D, F, with D as the weakest passing grade.

Oppgave 1

For every real number a we define the matrix $\mathbf{A}_a$ by

$$\mathbf{A}_a = \begin{pmatrix} 3 & 2 & -4 \\ 1 & 1 & 2a-3 \\ 2 & a & 2 \end{pmatrix}.$$

(a) Find the determinant $|\mathbf{A}_a|$.

(b) For what values of a does the equation system

$$\begin{aligned} 3x + 2y - 4z &= 2 \\ x + y + (2a-3)z &= 3 \\ 2x + ay + 2z &= 6 \end{aligned}$$

have (i) exactly one solution, (ii) several solutions, (iii) no solutions?

Oppgave 2

Find the general solution of the differential equation

$$\dot{x} - x = \frac{e^t}{t}, \qquad t > 0.$$

Also find the particular solution that gives $x = e^{-1}$ for $t = 1$.

(Cont.)

Oppgave 3

The equation system

$$x + e^{v-u} - \ln y = 1$$
$$xy - u + 2v^2 = e$$

defines u and v as continuously differentiable functions of x and y around the point $(x, y, u, v) = (1, e, 0, 0)$. (You are not supposed to prove this.)

(a) Differentiate the system (i.e. calculate differentials).

(b) Find a general expression for v'_y.

Oppgave 4

In this problem A, a, and b are constants, with $A > 0$ and $a \neq 0$. The equation

$$u + \ln u = Ax + \frac{1}{2}y^2$$

defines u implicitly as a function $u = u(x, y)$ of x and y. (You are not supposed to prove this.)

(a) Find expressions for the partial derivatives $u'_1(x, y)$ and $u'_2(x, y)$.

(b) Use Lagrange's method to set up the necessary first-order conditions for a point (x, y) to solve the problem

(P) maximize/minimize $ax + by$ subject to $u(x, y) = K$,

where K is a constant. Show that there exists exactly one point (x^*, y^*) that satisfies these conditions.

(c) Show that the level curve $u(x, y) = K$ consists of all points (x, y) for which $y = \pm\sqrt{Q - 2Ax}$, where Q is a constant that depends on K.

(d) (Difficult.) Use the result from part (c) to decide for what values of a and b the point (x^*, y^*) that you found in part (b) is a maximum point in problem (P). (*Hint:* Draw the level curve $u(x, y) = K$ together with some level curves for $ax + by$.)