

ECON3120/4120 – Mathematics 2, fall term 07: **Problems for seminar 2, Sep. 10**

1 Find the following limits:

$$(a) \lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2}{3x^3} \quad (b) \lim_{x \rightarrow 7} \frac{\sqrt[3]{x+1} - \sqrt{x-3}}{x-7}$$

2 The following system of equations defines  $u = u(x, y)$  and  $v = v(x, y)$  as  $C^1$  functions of  $x$  and  $y$  around the point  $P$  where  $(x, y, u, v) = (1, 1, 1, 1)$ :

$$\begin{aligned} 2uv + v^2 &= 2x + y \\ u - v &= x^2 - y^2 \end{aligned}$$

(a) Differentiate the system. Then find the values of  $\partial u/\partial x$ ,  $\partial u/\partial y$ ,  $\partial v/\partial x$  and  $\partial v/\partial y$  at the point  $P$ .

3 Assume that the marginal cost function of a firm is

$$C'(x) = x^2 + x - 10$$

and that the fixed costs are 50. Find the cost function.

4 Find the integrals:

$$(a) \int \frac{x}{1+x^2} dx \quad (b) \int_0^1 (1-2x)^{50} dx \quad (c) \int_1^{e^2} \sqrt{x} \ln x dx$$

5 Evaluate  $\int_0^2 2x^2(2-x)^2 dx$ . Give a rough check of the answer by sketching the graph of  $f(x) = 2x^2(2-x)^2$  over  $[0, 2]$ .