

ECON3120/4120 – Mathematics 2, fall term 07: **Problems for seminar 6, Oct. 22**

**1** (Exam problem 30/5-05)

(a) Calculate the determinant of  $\mathbf{A}_t = \begin{pmatrix} 0 & t & 1 \\ 4 & -2 & 8 \\ 1 & 1 & 1 \end{pmatrix}$

(b) Find  $x, y$  and  $z$  such that

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} - \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix}$$

**2** Exam problem 142.

**3** Find the general solution of the differential equation

$$\dot{x} + \frac{2}{t}x = e^t$$

Find, in particular, the integral curve passing through  $(t, x) = (1, 1)$ .

**4** Given the matrix

$$\mathbf{A}_t = \begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$$

(a) Calculate  $|\mathbf{A}_t|$  and show that  $\mathbf{A}_t^{-1}$  exists for all  $t$ .

(b) Show that for a certain value of  $t$  we have  $\mathbf{A}_t^3 = \mathbf{I}_3$ , where  $\mathbf{I}_3$  is the identity matrix of order 3.

(c) Find the inverse of  $\mathbf{A}_1$ .

(d) Suppose that  $\mathbf{A}$  and  $\mathbf{B}$  are invertible  $n \times n$ -matrices. Show that if  $\mathbf{A}'\mathbf{A} = \mathbf{I}_n$ , then  $(\mathbf{A}'\mathbf{B}\mathbf{A})^{-1} = \mathbf{A}'\mathbf{B}^{-1}\mathbf{A}$ .