University of Oslo
Department of Economics

Voluntary term paper in ECON3120/4120 Mathematics 2

Announced: Monday 10 November 2008.

To be handed in during the lecture on Friday 14 november 2008.

If you can't manage the deadline, you can hand your paper in at the department reception, 12th floor, but the later you hand it in, the longer you will have to wait for it to be marked.

Further instructions:

- This term paper is **voluntary** and is meant to give you a little practice in writing answers to mathematics problems.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- The problems are given only in English, but you are free to write your answers in Norwegian, Danish, or Swedish.)
- Please write your answers as clearly as possible, giving reasons for your statements. They should be written in such a way that another student can understand them. And please write legibly!
- If you happen to recognize a problem as one you have seen before, don't look up the old answer, but do the problem as if you haven't seen it before.

Name:

Problem 1

Find the integrals

(a) $\displaystyle\int_e^{e^2} \frac{1}{x \ln x}\, dx.$

(b) $\displaystyle\int_0^4 \Big(\frac{2}{\sqrt{x}+1} + \frac{1}{2}\sqrt{x}\Big)\, dx.$

Problem 2

Find the solution of the differential equation $\dot{x} = t^3 e^{-x}\sqrt{t^2+1}$ that passes through the point $(t_0, x_0) = (0, 0)$.

(Cont.)

Problem 3

The point $(x_0, y_0) = (1, 0)$ lies on the curve

$$xe^y - y^2 = 2x^3 - e^{x-1}.$$

Show that $(x_1, y_1) = (0, -4)$ lies on the tangent to the curve at (x_0, y_0).

Problem 4

Let f be a function of two variables, given by

$$f(x, y) = (x^2 - axy)e^y$$

where $a \neq 0$ is a constant.

(a) Find the stationary points of f and decide for each of them if it is a local maximum point, a local minimum point or a saddle point.

(b) Let (x^*, y^*) be the stationary point where $x^* \neq 0$, and let $f^*(a) = f(x^*, y^*)$. Find $df^*(a)/da$. Show that if we let $\hat{f}(x, y, a) = (x^2 - axy)e^y$, then

$$\hat{f}_3'(x^*, y^*, a) = \frac{df^*(a)}{da}$$

Problem 5

Consider the problem

$$\max\ 4 - \tfrac{1}{2}x^2 - 4y \quad \text{subject to} \quad 6x - 4y \leq a$$

(a) Write down the Kuhn–Tucker conditions.

(b) Solve the problem.

(c) With $V(a)$ as the value function, verify that $V'(a) = \lambda$, where λ is the Lagrange multiplier in (b).

Problem 6

(a) Find the determinant of $\mathbf{A} = \begin{pmatrix} a & a-1 & a \\ a-1 & 1 & 0 \\ a & 0 & a \end{pmatrix}$.

(b) For what values of a and b will the equation system $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b \\ 0 \\ 1 \end{pmatrix}$ have infinitely many solutions?

(c) For what values of a does there exist a matrix $\mathbf{B}$ such that $\mathbf{AB} = \mathbf{A} + \mathbf{B}$?