

## ECON3120/4120 Mathematics 2, autumn 2008

### Problems for Seminar 3, 15–19 September

1 Problem 63 in the exam problem booklet:

(a) The equation

$$3xe^{xy^2} - 2y = 3x^2 + y^2$$

defines  $y$  as a differentiable function of  $x$  around the point  $(x^*, y^*) = (1, 0)$ . Find the slope of the graph at this point by implicit differentiation. What is the linear approximation to  $y$  around  $x^* = 1$ ?

(b) In an equilibrium model the following system of equations is studied:

$$\begin{aligned} pF'(L) - r &= 0 \\ pF(L) - rL - B &= 0 \end{aligned} \quad (*)$$

where  $F$  is a twice differentiable function with  $F'(L) > 0$  and  $F''(L) < 0$ . All the variables are positive. Consider  $r$  and  $B$  as exogenous and  $p$  and  $L$  as endogenous variables, so that  $p$  and  $L$  are functions of  $r$  and  $B$ . Find expressions for  $\partial p/\partial r$ ,  $\partial p/\partial B$ ,  $\partial L/\partial r$ , and  $\partial L/\partial B$  by implicit differentiation.

(c) Determine, if possible, the signs of these partial derivatives. Show, in particular, that  $\partial L/\partial r < 0$ .

2 Problem 105 in the exam problem booklet:

The equation

$$x^2y^3 + (y + 1)e^{-x} = x + 2 \quad (*)$$

defines  $y$  as a differentiable function of  $x$  around  $(x, y) = (0, 1)$ .

(a) Compute  $y'$  at this point.

(b) Show that the curve given by (\*) intersects the  $x$ -axis in exactly one place.

3 Calculate the integrals:

$$(a) \int (2x^3 + 6x - 8) dx \quad (b) \int \frac{\sqrt[3]{x^2} - 5\sqrt[4]{x}}{\sqrt{x}} dx \quad (c) \int_0^1 \frac{e^{3x} + e^x - 1}{e^{2x}} dx$$

4 Show that  $\int \sqrt{x^2 + 3} dx = \frac{1}{2}x\sqrt{x^2 + 3} + \frac{3}{2}\ln(x + \sqrt{x^2 + 3}) + C$ .

5 Evaluate  $\int_0^2 2x^2(2 - x)^2 dx$ . Make a rough check of the answer by sketching the graph of  $f(x) = 2x^2(2 - x)^2$  over  $[0, 2]$ .