University of Oslo
Department of Economics

English

ECON3120/4120 Mathematics 2

Friday 8 December 2006, 09.00–12.00.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given: A (best), B, C, D, F, with D as the weakest passing grade.

Problem 1

Let $f(x) = \frac{1}{2}x - \frac{1}{4}x^2 + 5\ln(x+2)$.

(a) What is the domain of definition of f? Compute $f'(x)$ and $f''(x)$.

(b) Find the extreme points of f, if any.

(c) How many solutions does $f(x) = 0$ have? Sketch the graph of f.

(d) Compute the integral $\int_0^4 \left(\frac{1}{2}x - \frac{1}{4}x^2 + 5\ln(x+2)\right) dx$.

Problem 2

(a) Use Lagrange's method to solve the problem

$$\text{maximize} \quad 24x - x^2 + 16y - 2y^2 \quad \text{subject to} \quad x^2 + 2y^2 = 44\,.$$

(b) Suppose we replace the constraint in part (a) by $x^2 + 2y^2 \leq 44$. Write down the necessary Kuhn–Tucker conditions for a point (x, y) to solve this new problem, and find all solutions of these conditions.

Problem 3

For each real number t, define the matrix $\mathbf{A}_t$ as $\mathbf{A}_t = \begin{pmatrix} 1 & 1 & 2 \\ -t & 3 & 2 \\ t & 1 & 2 \end{pmatrix}$.

(a) Compute $|\mathbf{A}_t|$ and $|(\mathbf{A}_2)^3|$.

(b) Show that for a suitable value of s the matrix $\mathbf{B} = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 1/2 & s \\ 2 & -1/4 & -5/4 \end{pmatrix}$ is the inverse of $\mathbf{A}_2$. (It is not a good idea to use the formula for the inverse.)

(Cont.)

Problem 4

The equation system

$$tx^2 + y = 2t - \tfrac{1}{2}$$
$$2\ln x + 3y = x + \ln(2y) + t - \tfrac{1}{2}$$

defines x and y as differentiable functions of t around the point $x = 1$, $y = \frac{1}{2}$, $t = 1$. Find dx/dt and dy/dt at this point.