Universitetet i Oslo / Økonomisk institutt / NCF

ECON3120/4120 – Mathematics 2, fall term 09: **Solutions for seminar 8, Oct. 28**

You requested solutions for exam problem 92.

Problem 92

(a) Put $F(x,y) = xe^{x^2y} + 3x^2 - 2y - 4$. We have

$$\frac{dy}{dx} = -\frac{F_1'(x,y)}{F_2'(x,y)} = -\frac{e^{x^2y} + 2x^2ye^{x^2y} + 6x}{x^3e^{x^2y} - 2}.$$

At $(x,y) = (1,0)$ we have $\frac{dy}{dx} = -\frac{e^0+0+6}{e^0-2} = 7$.

(b) Taking z-derivatives gives

$$\begin{aligned} \frac{dx}{dz}e^y + xe^y\frac{dy}{dz} + \frac{dy}{dz}f(z) + yf'(z) &= 0 \\ \frac{dx}{dz}g(x,y) + x\Big(g_1'(x,y)\frac{dx}{dz} + g_2'(x,y)\frac{dy}{dz}\Big) + 2z &= 0 \end{aligned}$$

(Note: We could have calculated differentials, but since there is only one free variable z, we can then just «divide by dz».) Rearranging,

$$\begin{aligned} e^y\frac{dx}{dz} + \big(xe^y + f(z)\big)\frac{dy}{dz} &= -yf'(z) \\ \big(g(x,y) + xg_1'(x,y)\big)\frac{dx}{dz} + \qquad xg_2'(x,y)\frac{dy}{dz} &= -2z \end{aligned}$$

This is a linear system with dx/dz and dy/dz as the unknowns. The determinant of the system is

$$\begin{aligned} D &= \begin{vmatrix} e^y & xe^y + f(z) \\ g(x,y) + xg_1'(x,y) & xg_2'(x,y) \end{vmatrix} \\ &= xe^y\big(g_2'(x,y) - g(x,y) - xg_1'(x,y)\big) - f(z)\big(g(x,y) - xg_1'(x,y)\big), \end{aligned}$$

and by Cramér's rule,

$$\frac{dx}{dz} = \frac{1}{D}\begin{vmatrix} -yf'(z) & xe^y + f(z) \\ -2z & xg_2'(x,y) \end{vmatrix} = \frac{1}{D}\big[-xyf'(z)g_2'(x,y) + 2z\big(xe^y + f(z)\big)\big]$$

and

$$\frac{dy}{dz} = \frac{1}{D}\begin{vmatrix} e^y & -yf'(z) \\ g(x,y) + xg_1'(x,y) & -2z \end{vmatrix} = \frac{1}{D}\big[-2e^yz + y\big(g(x,y) + xg_1'(x,y)\big)f'(z)\big].$$