University of Oslo / Department of Economics / Nils Framstad / November 8, 2011

# ECON3120/4120 Mathematics 2: Re the term paper

There were five problems: two differential equation problems, one nonlinear programming problem, one problem of finding-and-classifying stationary points, and one of finding an elasticity. Altogether it seems it was slightly more than a full exam problem set. A total of 16 papers were submitted.

**For the differential equations problems** (there were two of these):

- There are a few of you who certainly need to practice your integration skills. A few of you need to drill the basics of the exponential function.
- There were no separable differential equations: they were linear. But one was of the form $\dot{x} = ax + b(t)$ and the other was of the form $\dot{x} - x = b(t)$. Compared to the form used in the book, one of them moves the $x$ term to the RHS and the other has a minus sign on it. This is evidently enough to confuse quite a few of you – some of you happily calculate as if sign does not matter. Look, would you hire an economist who doesn't know the difference between *paying* an interest of 100/year or *receiving* an interest of 100/year?
  - If you want to use a formula, always make sure that you have brought the problem to the same form as assumed.
  - Equations in general: you can verify answers by evaluating the left hand side and the right hand side separately, to see that they match.
  - Those of you who use the integrating factor approach rather than the formula (that includes myself, so I don't have to look up all the time), will automatically get a verification of the sign when differentiating. But to those who use the formula because they do not want to learn why it works – which is OK in this course! – the integrating factor is maybe a bit too advanced.
- A typical error was to add a constant at the end. Not in the integral, simply at the end of the line. Obviously, this is a method failure (and again, it would be easily detected if you checked your answer).
- So, you have arrived at $\dot{y} = Ce^t - \frac{1}{b+1}e^{-bt}$ and you shall find $y$. Another integration, another … another arbitrary constant, or another instance of the «$C$» letter?
- Autumn 08 problem 4 (b) asks for a tangent. That is a line, not merely a slope.

**The classify stationary points problem:** The general impression is that this worked out fairly well theoretically, and most mistakes were calculations mistakes.

- Drill differentiation!
- Solving the nonlinear equations, could be a bit of work with many chances of making an erroneous calculation. But, assuming you have differentiated correctly: Once you have found what you think is a stationary point, you can easily verify it by inserting into the partial derivatives and see that you get zero.
  - A couple of odd approaches led to false solutions. The logical issue was subtle, and most certainly not an intentional trap.

**The Kuhn–Tucker problem:** lots of mistakes here:

- A very common mistake is the (false!) assumption that equality in the constraint implies strictly positive multiplier. A simple counterexample is $\max_{x\le 0} x^2$, where you of course choose the stationary point $x = 0$, with zero multiplier.
  - Any admissible (admissible = satisfying the constraints) stationary point, satisfies the Kuhn–Tucker conditions (with all multipliers being zero).
- Indeed, in this problem, the origin is a stationary point. But still a few of you failed to check (or point out!) that the multipliers were OK. That is, you got that they all had to be equal to each other, but not that there is a common value (necessarily zero, since $x + y < A^2$).
- Although I gave you a cookbook for checking all possible combinations of active/inactive constraints in order to find all possible solutions, that does not mean you are supposed to go ahead and solve – you were indeed not asked to find the solution! (The reason why I did not give a «solve the problem» question, was that this would be too hard and time-consuming – and I bet that those who actually tried, would probably be the first to agree.)
  In order to answer to part (b), you proceed as follows:
  - The problem text tells you that the solution point is not on the axis.
    - This implies that the respective multipliers, are both zero.
    - There is only one remaining multiplier. This is zero or nonzero, and we are asked to show the latter.
    - This could be done by showing that the case where it is zero, does not lead to any solution candidate (this is proof by contradiction).
  - Then there is a hint: show that off the axes, there is no stationary point.
    - Why this hint? The case we want to disprove, is where all multipliers are zero, which implies that we have a stationary point.
    - If we can prove that no such exists, we have proven that it is impossible to have all multipliers zero, and we are done.
    - Well, we are not precisely in that case: we are assuming we are off axes. So if we can prove that there are no stationary point off the axes, then we have proven that Kuhn–Tucker cannot hold off axes with all multipliers nonzero. The conclusion follows
  - Part of the reason why the problem was formulated this way, was to enable you to divide by $x$ and $y$ without worrying that they might be zero. (Thereby, I saved those students who would (and did) happily divide them out without worrying about zeroness, from a well-deserved penalty ... why ain't there a halo around my head? Oh, somebody used it to divide by.)

**The elasticity problem:** You seem to take logs and differentiate (which is basically what I would do), but then a few of you ran into problems. In this problem, the multiplicative structure makes both logs and elasticities way easier. The probably quickest solution is to write $\mathrm{El}_x(y^3/x^3) = 3\mathrm{El}_x y - 3\mathrm{El}_x x = 3\mathrm{El}_x - 3$, and then for the right hand side: $\mathrm{El}_x\left[(x+a)^p(y+b)^q\right] = p\mathrm{El}_x(x+a) + q\mathrm{El}_x(y+b)$. Now use the chain rule: $\mathrm{El}_x(y+b) = \mathrm{El}_y(y+b)\mathrm{El}_x y$, then calculate the elasticity of the sum, and solve for $\mathrm{El}_x y$ by gathering terms.