

ECON3120/4120 Mathematics 2

Tuesday 31 May 2011, 14:30–17:30.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1

For every real number t , let $\mathbf{A}_t = \begin{pmatrix} 4+t & 2 & 1 \\ 2 & t & 0 \\ 1 & 0 & t \end{pmatrix}$.

- (a) Show that $|\mathbf{A}_0| = 0$. Is $|\mathbf{A}_t| = 0$ for any other values of t ?
- (b) Find a necessary and sufficient condition on a , b , and c for the following system of equations to have at least one solution:

$$\begin{aligned} 5x + 2y + z &= a \\ 2x + y &= b \\ x + z &= c \end{aligned}$$

Problem 2

The equation $xy + y^3 = 3$ defines y implicitly as a function $y = \varphi(x)$ around the point $(x_0, y_0) = (2, 1)$.

- (a) Find an expression for $\varphi'(x)$.
- (b) Find the quadratic approximation to $\varphi(x)$ around $x_0 = 2$.

Problem 3

Consider the differential equation

$$\dot{x} + \frac{1}{2}x = 2 - t. \quad (*)$$

- (a) Find the general solution of equation (*).
- (b) The t -axis (i.e. the straight line $x = 0$) is tangent to the graph of one solution of (*). Find the point of tangency and the corresponding solution.

(Cont.)

Problem 4

Let F be the function defined by $F(t) = \int_1^t \frac{2 - \ln x}{x^3} dx$ for all $t > 0$.

- (a) Show that F has a maximum point and find the maximum value of F .
- (b) Find $\lim_{t \rightarrow \infty} F(t)$ if this limit exists.

Problem 5

Let f be the function defined by

$$f(x, y) = -xy^3 - xy^2 + y - x^2$$

and let S be the set $S = \{(x, y) : x > 0, xy \geq 1\}$.

- (a) Show that f has no stationary point in S .
- (b) Find the maximum point or points of $f(x, y)$ over S . You may assume that f has a maximum value over S .