University of Oslo
Department of Economics

English version

ECON3120/4120 Mathematics 2

Thursday 4 December 2003, 14.30–17.30

There are 2 pages of problems.

All written or printed material may be used, as well as pocket calculators.

State reasons for all your answers.

The grades given run from A to F, with A as the best grade
and E as the weakest passing grade.

Problem 1

Consider the function f defined for all x by

$$f(x) = (ax+1)e^{-bx}, \quad a > b > 0$$

(a) Compute $f'(x)$ and $f''(x)$.

(b) Examine where f is increasing and where f is decreasing. Show that f has a maximum point x^*, and find this point. Show that $x^{**} = x^* + 1/b$ is an inflection point for f.

(c) Examine the limits of $f(x)$ when x approaches ∞ and when x approaches $-\infty$.

(d) Compute $\int_0^\infty f(x)\,dx$.

Problem 2

The equation

$$ze^z - xy = 0$$

defines z as a function of x and y in a neighbourhood of the point $(x,y,z) = (1,e,1)$. Find $z_1'(1,e)$, $z_2'(1,e)$, and $z_{12}''(1,e)$.

Problem 3

A certain model of wage formation contains the integral $\int \frac{w}{(1-w)^3}\,dw$. Compute this integral.

(Cont.)

Problem 4

Consider the matrix $\mathbf{A} = \begin{pmatrix} 11 & -6 \\ 18 & -10 \end{pmatrix}$.

(a) Compute $|\mathbf{A}|$. Show that for a suitable number c we have $\mathbf{A}^2 + c\mathbf{A} = 2\mathbf{I}_2$.

(b) Show that there is no 2×2 matrix $\mathbf{B}$ such that $\mathbf{B}^2 = \mathbf{A}$.

Problem 5

Solve the following problem by means of Lagrange's method:

$$\text{minimize} \quad \ln(2 + x^2) + y^2 \quad \text{subject to} \quad x^2 + 2y = 2$$

(You can take it for granted that the minimum value exists.)

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