University of Oslo
Department of Economics

English

ECON3120/4120 Mathematics 2

Thursday 10 December 2009, 09:00–12:00.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given: A (best), B, C, D, E, F, with E as the weakest passing grade.

Problem 1

(a) For what values of x does the following matrix have an inverse?

$$\mathbf{A} = \begin{pmatrix} x+3 & 0 & 2 \\ 0 & 4-x & 3 \\ 0 & 4 & -x \end{pmatrix}$$

(b) Find a matrix $\mathbf{B}$ such that $\mathbf{B}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 2x - y + w \\ x - y + 2z \end{pmatrix}$.

(*Hint:* What must be the order of $\mathbf{B}$?)

(c) Find the matrix $\mathbf{C}$ when $(\mathbf{C}^{-1} - 2\mathbf{I}_2)' = -2\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$.

Problem 2

(a) Use Lagrange's method to solve the problem

$$\max\ xy \quad \text{subject to} \quad (x+2a)(y+3a) = A\,.$$

The constants a and A are positive, with $a^2 < \frac{1}{6}A$.

(b) Denoting the optimal values of x and y by x^* and y^*, compute the value function $f^*(a, A) = x^*y^*$ and its partial derivatives with respect to A and a.

(c) Compare the results in part (b) with the values you find by using the envelope theorem.

(Cont.)

Problem 3

The following system defines u and v as differentiable functions of x and y in a neighbourhood of $P = (x, y, u, v) = (1, 1, 0, 1)$,

$$u - v^2 - 2x - y^2 = -4$$
$$e^{xu} + e^{yv} = 1 + e$$

(a) Differentiate the system and express the differentials of u and v in terms of the differentials of x and y.

(b) Find the partial derivatives of v with respect to x and y at P.

(c) Estimate the value of $v(0.99, 1.02)$.

Problem 4

(a) (In the integral below, k and r are constants and x is positive.)

(i) Show that for $r \neq -1$ we have

$$\int (x + kx^{-r})^{-1}\, dx = \frac{\ln |k + x^{r+1}|}{r+1} + C\,.$$

(Recall that $\frac{d}{du}(\ln |u|) = \frac{1}{u}$.)

(ii) What happens to the integral when $r = -1$? For what values of k, if any, will you get the same expression for $r = -1$ as for $r \to -1$?

(b) Use (a) (i) to find the general solution of the differential equation

$$\dot{x} = 2(x - x^{2-e})t/(e - 1),$$

where $e \approx 2.71828$ is the base number of the natural exponential function.

(c) Find the particular solution that passes through (e, e) and the particular solution that passes through $(1, 1)$.