University of Oslo
Department of Economics

English

ECON3120/4120 Mathematics 2

Tuesday 2 June 2009, 14:30–17:30.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1

The function f is defined over the entire xy-plane by

$$f(x,y) = e^{3x} + 3ye^{x} - y^3\,.$$

(a) Find the first and second order partial derivatives of f.

(b) Find the stationary points of f, if any, and determine whether they are local maximum points, local minimum points or saddle points.

(c) The level curve $f(x,y) = 3$ passes through the point $(x,y) = (0,-2)$. Find an equation for the tangent to the level curve at this point.

Problem 2

Let $f(x) = x^2e^x$ for all x.

(a) Over which one of the intervals $I_1 = (-\infty,-2)$, $I_2 = (-\infty,0)$, and $I_3 = (-2,\infty)$ does f have an inverse function?

(b) Let g be the inverse function of f and let x_0 be a point where $f'(x_0) \neq 0$. Find an expression for $g'(f(x_0))$.

(Cont.)

Problem 3

(a) Use Gaussian elimination to find a necessary and sufficient condition for the linear equation system

$$\begin{aligned} x + y - 3z &= a \\ x - 3y + 4z &= b \\ 3x - y - 2z &= c \end{aligned}$$

to have at least one solution.

(b) Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & 5 \\ r & 3 & -1 \\ 1 & s & 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{pmatrix} 2 & t & -19 \\ 1 & -4 & u \\ -1 & 5 & 11 \end{pmatrix}.$$

Calculate the matrix product $\mathbf{AB}$. If $\mathbf{B} = \mathbf{A}^{-1}$, what are the values of r, s, t, and u?

Problem 4

(a) Find the integral $\displaystyle\int \frac{t+1}{t(1+te^t)}\,dt$.

($\textit{Hint:}$ Try the substitution $u = 1 + te^t$.)

(b) Find the general solution of the differential equation

$$t(1+te^t)\dot{x} = x^2(1+t)\,. \tag{$*$}$$

(c) The differential equation $(*)$ has a solution curve that passes through $(1,1)$. Find an equation for the tangent to this solution curve at that point.