33

* At the Thursday 10^{15} seminar, someone requested a Kuhn–Tucker based solution. Kristoffer has posted one such.

* This one does not use Kuhn-Tucker.

* Recall that the problem says
  "Find the minimum value [...] taking it for granted that there is a minimum".
  - → you have the choice of method
  - → you do not need to prove existence! (But, this note will indeed.)

* P4 corrects an <u>error</u> from the Thu 10^{15} session

We have $$f(x,y) = u(x) + v(y)$$

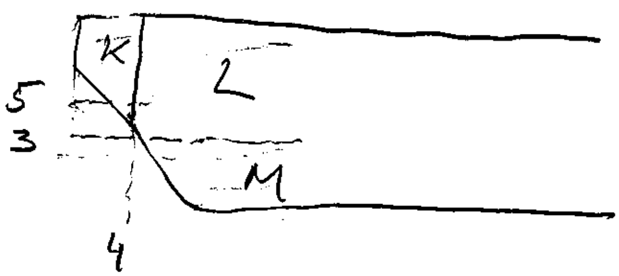
where $$u(x) = x^2 - 8x$$
$$v(y) = -\frac{1}{3}y^3 + 4y^2 - 15y$$

Derivatives: $$u'(x) = 2\cdot(x-4)$$
$$v'(y) = -y^2 + 8y - 15$$

which [calculate the zeroes]

$$= -(y-3)(y-5)$$

Looking at the shape of A:



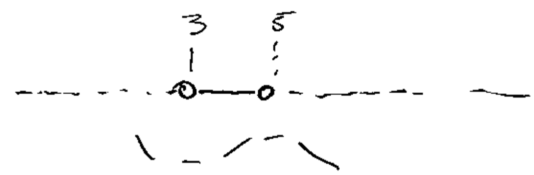
* No min. in the (open) set K

$$f(x,y) > f(4,y)$$

* No min in the (open) sets L & M

f smaller on than to the right

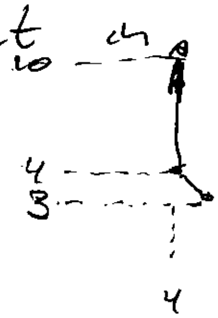
* No min for $y < 3$ either (i.e. none on the "\" boundary of M)

v' : 

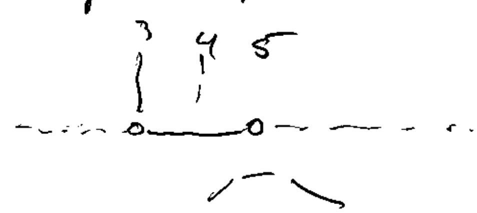
Choosing $y = 3$ improves over $y < 3$

Choosing $x = 5$ improves over $x > 5$

$(5,3)$ is the up-left corner of M

So for each point in A, there is some point on  which improves

(and that set is compact, so EVT $\Rightarrow$ min exists)

· Case $x = 4$. $v'(y)$: 

For min, no $y \in (4,10)$ is possible.

Candidates: $(4,4)$ and $(4,10)$

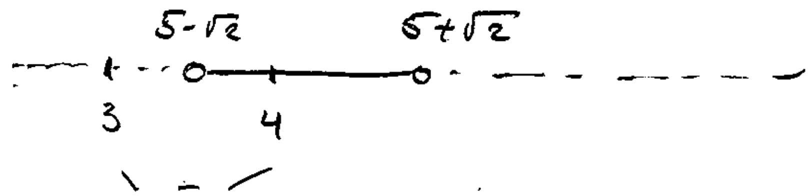
· Case \ $x + y = 8$, $x \in [4,5]$ or equiv.: $y \in [3,4]$

$$f(8-y, y) = u(8-y) + v(y) =: g(y).$$

$$g'(y) = -u'(8-y) + v'(y)$$
$$= -2(8-y-4) - y^2 + 8y - 15$$
$$= -y^2 + 10y - 23$$
$$y = 5 \pm \sqrt{25 - 23}$$

Note: $5 - \sqrt{2}$ is $\in [3, 4]$.

$g'(y)$



Smallest at $y = 5 - \sqrt{2}$; in particular $(x, y) = (4, 4)$ can be ruled out

Remaining candidates:

$(3 + \sqrt{2},\ 5 - \sqrt{2})$ and $(4, 10)$

$f \approx -33.6$ $\qquad$ $\underline{\underline{f = -\frac{298}{3}}}$