Mathematics 2

Introduction :

- → what you will learn
- → what you should know already
- → practicalities

✱ Books:

- English <u>or</u> Norwegian.
  - ↳ M2 : EMEA
  - ↳ M3 : EMEA
    (+ bits...)

  - ↳ M2 : MA1 + LA + MA2 for 1 kap
  - ↳ M3 : LA + MA2

✱ Teaching language

- English...

Topics (non-exhaustive list!)

→ Optimization (max/min)
  → under eq./ineq. constraints

- You should already know
  → 1st & 2nd order cond's
    in one variable (and have
    seen it in two, with Lagrange)
  → concavity/convexity, 1 var.

... but will be covered
from scratch

M2

→ Integrals: You know $f'(x)$,
find $f(x)$

• Should know: differentiation

→ Differential equations: e.g. find the functions $x(t)$ s.t.

$$x'(t) = r\,x(t).$$

[Solution: $C\,e^{rt}$]

M 2

→ Linear algebra

→ Vector & matrix notation

→ Vector & matrix algebra

→ Linear eq. systems

→ Applications to <u>linear ODE</u>

eq. systems : e.g.

$$\begin{cases} F(x, y, u(x,y), v(x,y)) = C \\ G(x, y, u(x,y), v(x,y)) = D \end{cases}$$

Find $u'_x(x,y)$

Some notation / terminology

$\mathbb{R}$: the real numbers

$\mathbb{N}$: the natural numbers: $\{1, 2, 3, 4 \ldots\}$

(... except in French, which obeys ISO 31-11: $\{0, 1, 2, 3 \ldots\}$)

There are also $\mathbb{Z}$ = integers, $\mathbb{Q}$ = rationals

C^n : The functions whose derivatives up to & including order n, exists and are continuous.

($C^n \subsetneq C^{n+1}$!)

Intervals: (a, b) does not include a or b

$[a, b)$ includes a, not b

note: $[a, +\infty)$, $(a, +\infty)$. Infinity is not a number

Closed / open set: includes every / resp: no boundary point

Vectors

(M2: 3200/4200 will use this ; M3: M3 will use this)

"Vectors", to mathematicians:

"objects you can scale and add"

- to M2/M3: ordered n-tuples

  (pairs, triplets, quadruples ...)

  e.g. $(1, 3, -4, 2.1, -0.4)$

  separators: commas; decimal point

  or $(\frac{1}{2}, -\frac{1}{3})$ etc.

  ("ordered": $(1, 2) \neq (2, 1)$)

Ex: price list

Vector notation, distinguishing them from numbers

$\vec{x}$ common in physics

$\bar{x}$ ..... non-physics

$\underset{\sim}{x}$ statistics

$\mathbf{x}$ (boldfaced): textbooks

$\mathbb{x}$ ("blackboard bold") mimicking textbooks

$x \in \mathbb{R}^4$: mathematicians often

"4 copies of the real line"

do not use any particular font (rather, they specify)

Matrices: same, but upper-case.

You are free to choose among these!

(But you are better off being consistent!)

$\bar{x} = (x_1, \ldots, x_n)$

↑ vector; $x_1, \ldots, x_n$: numbers

Book: $x^{(1)}$, $x^{(2)}$ are two vectors

Function f of several variables: $f(\bar{x})$

(M3 will use this!)

So: the followings are the same:

$f(x,y,z)$ $\quad f(x_1, x_2, x_3)$ $\quad f(\bar{x})$, $\bar{x} \in \mathbb{R}^3$

We gather the partial derivatives in one vector:

∇f denotes $(f'_1, \ldots, f'_n)$.

(the "gradient" of f).

There are row vectors: $(1, 0, -4)$

and column vectors: $\begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}$

As long as one has not introduced matrices, the choice of row notation vs column notation is just a matter of convenience.

M2: rows until matrices are introduced

M2 from then on, and M3: columns unless otherwise specified

→ ∇f always a row!

3200/4200 etc: follow that course's convention.

These four vectors are bookkeeping tools

We can do calculations too.

"dot"

$$\bar{p} \cdot \bar{q} = p_1 q_1 + p_2 q_2 + \dots + p_n q_n$$

$\bar{p}$: price list

$\bar{q}$: quantities bought

$= $ total cost

E.g. $(\$1/\text{newspaper}, \$2/\ell \text{ gasoline}, -\$1/\text{bag of waste}) \cdot (4 \text{ newspapers}, \frac{15}{4}\ell \text{ gasoline}, -2 \text{ bags of waste})$

$= \$4 + \$\frac{15}{2} + \$2 = \underline{\$9.50}$

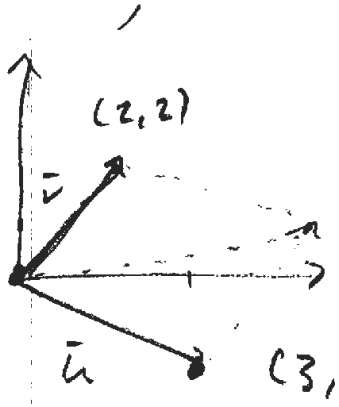
Vector addition: $\bar{u}$ $\bar{v}$

$$\bar{u} = (u_1, \ldots, u_n)$$

$$\bar{v} = (v_1, \ldots, v_n)$$

$$\bar{u} + \bar{v} = (u_1 + v_1, \ldots, u_n + v_n)$$

Geometric



$\bar{u} + \bar{v} = (5, 1)$

Length: $\|\bar{v}\| = \sqrt{v_1^2 + \ldots + v_n^2}$ (Pythagoras)