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(a)

$$El_x\left(y^2 e^{x+\frac{1}{y}}\right) = 0$$

$$El_x y^2 + El_x e^x + El_x e^{\frac{1}{y}} = 0$$

$$2 El_x y + x + \frac{1}{y} El_x\left(\frac{1}{y}\right) = 0$$

$$2 El_x y + x + \frac{1}{y}\left(\underbrace{El_x 1}_{=0} - El_x y\right) = 0$$

$$\left(2 - \frac{1}{y}\right) El_x y + x = 0$$

$$El_x y = \frac{-x}{2 - \frac{1}{y}}$$

$$= \frac{xy}{1-2y}$$

(67)

(b) Differentiate:

$$\alpha u^{\alpha-1}\,du + \beta v^{\beta-1}\,dv = 2^{\beta}\,dx + 3y^{2}\,dy$$
$$\alpha u^{\alpha-1}v^{\beta}\,du + \beta u^{\alpha}v^{\beta-1}\,dv - \beta v^{\beta-1}\,dv = dx - dy.$$

At $P: (x,y,u,v) = (1,1,1,2)$.

$$\alpha\,du + \beta 2^{\beta-1}\,dv = 2^{\beta}\,dx + 3dy \quad (1)$$
$$\left.\begin{aligned} \alpha 2^{\beta} du + \cancel{\beta 2^{\beta-1}}\,dv - \cancel{\beta 2^{\beta-1}}\,dv &= dx - dy. \\ \hookrightarrow \qquad \alpha 2^{\beta} du &= dx - dy \end{aligned}\right\} \quad (2)$$

$$(2): \quad du = \frac{2^{-\beta}}{\alpha}\,dx - \frac{2^{-\beta}}{\alpha}\,dy.$$

Insert du from (2) into (1). Result:

$$\alpha\left(\frac{2^{-\beta}}{\alpha}\,dx - \frac{2^{-\beta}}{\alpha}\,dy\right) + \beta 2^{\beta-1}\,dv = 2^{\beta}\,dx + 3\,dy.$$
$$\beta 2^{\beta-1}\,dv = (2^{\beta} - 2^{-\beta})\,dx + (3 + 2^{-\beta})\,dy$$
$$dv = \frac{2^{\beta} - 2^{-\beta}}{\beta 2^{\beta-1}}\,dx + \frac{3+2^{-\beta}}{\beta 2^{\beta-1}}\,dy.$$

Hence, $\dfrac{\partial u}{\partial x} = \dfrac{2^{-\beta}}{\alpha}$, $\dfrac{\partial u}{\partial y} = -\dfrac{2^{-\beta}}{\alpha}$, $\dfrac{\partial v}{\partial x} = \dfrac{2^{\beta} - 2^{-\beta}}{\beta 2^{\beta-1}}$, $\dfrac{\partial v}{\partial y} = \dfrac{3 + 2^{-\beta}}{\beta 2^{\beta-1}}$

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(c)

$$U(0.99, 1.01) \approx U(1,1) + U'_x(1,1)\cdot \Delta x + U'_y(1,1)\,\Delta y$$

$$= 1 + \frac{2^{-\beta}}{\alpha}\cdot\left(-\frac{1}{100}\right) + \frac{-2^{-\beta}}{\alpha}\cdot\frac{1}{100}$$

$$= 1 + \frac{-2^{-\beta} \mp 2^{-\beta}}{100\alpha}$$

$$= 1 - \frac{2^{-\beta} + 2^{-\beta}}{100\alpha}$$

$$= 1 - \frac{2\cdot 2^{+\beta}}{100\alpha}$$

$$= 1 - \frac{2^{1-\beta}}{100\alpha}$$

$\boxed{113}$

$$El_x\left(\frac{y^3}{x^3}\right) = El_x\left[(x+a)^p(y+b)^q\right]$$

$$El_x\, y^3 - El_x\, x^3 = El_x(x+a)^p + El_x(y+b)^q$$

Chain rule

$$3El_x\, y - 3El_x\, x = pEl_x(x+a) + qEl_x(y+b)$$

$$3El_x\, y - 3 = p\cdot\frac{d\ln(x+a)}{d\ln x} + q\cdot\frac{d\ln(y+b)}{d\ln y}\cdot El_x\, y$$

$$3El_x\, y - 3 = p\cdot\frac{\frac{1}{x+a}}{\frac{1}{x}} + q\cdot\frac{\frac{1}{y+b}}{\frac{1}{y}}\cdot El_x\, y$$

$$3El_x\, y - 3 = \frac{px}{x+a} + \frac{qy}{y+b}El_x\, y$$

$$\left(3-\frac{qy}{y+b}\right)El_x\, y = \frac{px}{x+a} + 3$$

$$El_x\, y = \frac{3+\frac{px}{x+a}}{3-\frac{qy}{y+b}}$$