$\boxed{86}$

$$f(x,y,z) = x^2 + x + y^2 + z^2$$

(a) $$\mathcal{L} = x^2 + x + y^2 + z^2 - \lambda(x^2 + 2y^2 + 2z^2 - 16)$$

(1) $$\mathcal{L}'_x = 2x + 1 - 2\lambda x = 0$$

(2) $$\mathcal{L}'_y = 2y - 4\lambda y = 0$$

(3) $$\mathcal{L}'_z = 2z - 4\lambda z = 0$$

(2): $2y(1-2\lambda) = 0$

(3): $2z(1-2\lambda) = 0$.

$\Rightarrow$ $\lambda = \frac{1}{2}$ or $y = z = 0$.

Case A. $\lambda = \frac{1}{2}$

From (1): $2x + 1 - x = 0$. $\Leftrightarrow$ $x = -1$

Constraint: $1^2 + 2y^2 + 2z^2 = 16$.

$\Leftrightarrow$ $y^2 + z^2 = \frac{15}{2}$.

86

(14)

(a) Candidate point:

$$(x,y,z) = (-1, y, z) \quad \text{with} \quad y^2 + z^2 = \frac{15}{2}.$$

Case B: $y = z = 0$

Constraint: $x^2 = 16$

$x_1 = 4$

$x_2 = -4.$

Candidate points $(x,y,z) = (4,0,0)$ $\lambda = \frac{9}{8} \geq 0$

$(x,y,z) = (-4,0,0).$ $\lambda = \frac{7}{8} \geq 0.$

Evaluate to find max/min.

$f(-1,x,y)$ with $y^2 + z^2 = \frac{15}{2}$ $= 1 - 1 + \frac{15}{2} = \frac{15}{2}$

min

$f(4,0,0) = 20$ max

$f(-4,0,0) = 12$

86

(b) Either solve by Kuhn-Tucker.

Or note that extreme points must be on the surface $x^2 + 2y^2 + 2z^2 = 16$, or inside the ball $x^2 + 2y^2 + 2z^2 < 16$.

In the latter case, it must be a stationary interior point.

Stationary point:

$$\left.\begin{aligned} f'_x &= 2x+1 &= 0 \\ f'_y &= 2y &= 0 \\ f'_z &= 2z &= 0 \end{aligned}\right\} \quad (x,y,z) = \left(-\tfrac{1}{2}, 0, 0\right).$$

$$f\left(-\tfrac{1}{2}, 0, 0\right) = -\frac{1}{4} \qquad \text{Minimum.}$$

Maximum is thus $(x,y,z) = (4,0,0)$.