Seminar 1

P.18

$$By\ L'Hôpital's\ rule \qquad \lim_{x\to 0}\frac{e^x-1-x}{x\sqrt{1+x}-x}=\frac{0}{0}=\lim_{x\to 0}\frac{e^x-1}{\sqrt{1+x}+\frac{1}{2}x(1+x)^{-\frac{1}{2}}-1}=\frac{0}{0}$$

$$=\lim_{x\to 0}\frac{e^x}{\frac{1}{2}(1+x)^{-\frac{1}{2}}+\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{4}x(1+x)^{-\frac{3}{2}}}=\frac{1}{1}=1.$$

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$$By\ L'Hôpital's\ rule \qquad \lim_{x\to 1}\frac{lnx-x+1}{(x-1)^2}=\frac{0}{0}=\lim_{x\to 1}\frac{\frac{1}{x}-1}{2(x-1)}=\frac{0}{0}=\lim_{x\to 1}\frac{-x^{-2}}{2}=-\frac{1}{2}.$$

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a)

$$f'(x)=\frac{e^{x-3}}{2+e^{x-3}}>0\to f(x)\ is\ strictly\ increasing.$$

$$\lim_{x\to-\infty}f(x)=\ln 2.$$

$$\lim_{x\to+\infty}f(x)=+\infty.$$

$$So\ the\ range\ of\ f(x)\ is\ (ln2,+\infty)$$

b)

$$x=\ln\left(2+e^{g(x)-3}\right).$$
$$e^x=2+e^{g(x)-3}.$$
$$e^{g(x)-3}=e^x-2.$$
$$g(x)-3=\ln(e^x-2).$$
$$g(x)=\ln(e^x-2)+3.$$

Where is g difined?

$$e^x-2>0\to e^x>2\to x>ln2.$$
$$g\ is\ defined\ for\ all\ x>ln2.$$

c)

$$f'(x)=\frac{e^{x-3}}{2+e^{x-3}}\to f'(3)=\frac{1}{2+1}=\frac{1}{3}.$$
$$f(3)=\ln(2+1)=ln3.$$
$$g'(x)=\frac{1}{e^x-2}e^x.$$
$$g'\left(f(3)\right)=g'(ln3)=\frac{e^{ln3}}{e^{ln3}-2}=\frac{3}{3-2}=3.$$
$$easy\ to\ see\ that \qquad f'(3)=\frac{1}{g'\left(f(3)\right)}.$$