Have:

vectors (row, today) in $\mathbb{R}^n$

$\vec{u} = \vec{v}$ same n

$\cdot \vec{v}$

$\vec{u} + \vec{v}$ same n

$\vec{u} \cdot \vec{v}$

For $n = 2$ (& 3): graphical representation (interpretation) of (equality,) scaling & addition [and thus subtraction]

and a bit of understanding of $\bullet$

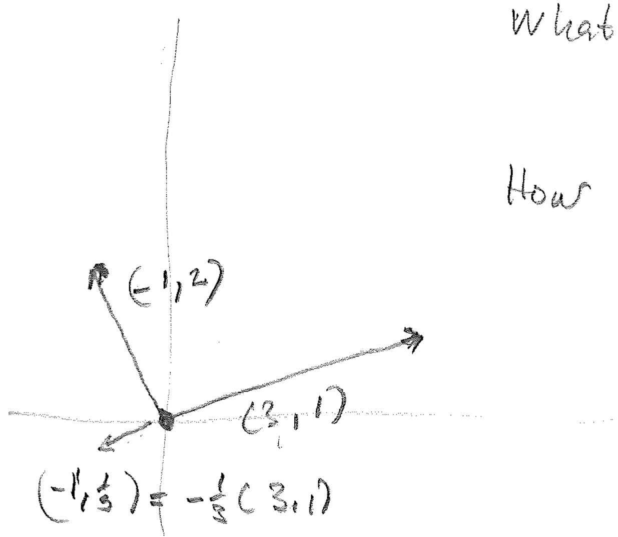
What do we mean by $=$? (same length, same direction)

How to add? (put head-to-toe)

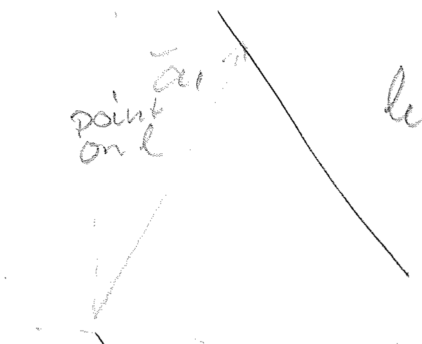
Does the dot product of of relate to anything?

($=0$? >0? ...)

$\perp$

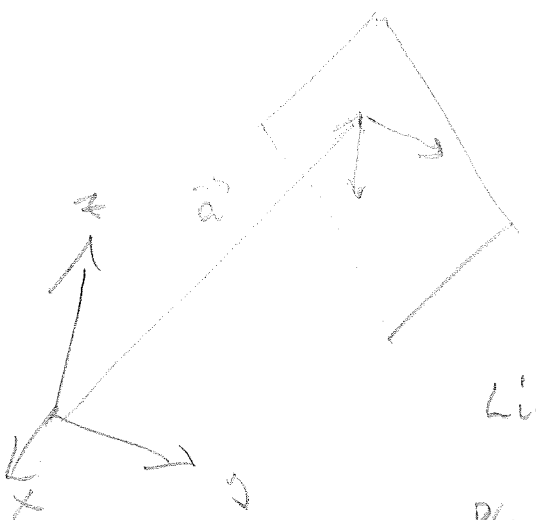


Lines, planes, hyperplanes



line l = points that can be written $\vec{a} + t\vec{v}$, some number t.

Two points $\vec{a} \neq \vec{b}$? $\vec{v} = \vec{b} - \vec{a}$.



Plane

$\vec{a} + s\vec{u} + t\vec{v}$,

$s \in \mathbb{R}$, $t \in \mathbb{R}$

Line: "one-dimensional" (the "t")

Plane: two-dim (s & t)

But wait.......

Line: 1-dim (linear) object in
$\mathbb{R}^2$ ... or $\mathbb{R}^3$ ... or $\mathbb{R}^n$
$\vec{a} + t\vec{v}$ works in $\mathbb{R}^n$ too.

Plane: 2-dim in $\mathbb{R}^3$?
But also: n-1 - dim in $\mathbb{R}^n$!
Instead of specifying the n-1 directions
(the two, $\vec{u}$ & $\vec{v}$ on the figure) we
can specify the n^{th} ("3rd");
(a $\vec{p}$ s.t. $\vec{p} \cdot \vec{u} = \vec{p} \cdot \vec{v} = 0$ in $\mathbb{R}^3$).

Hyperplane: $S \ni \vec{a}$;
$$\vec{p} \cdot (\vec{x} - \vec{a}) = 0$$
where $\vec{p} \perp$ the plane.

⟶ Budget! ⟵

Q: have you seen this for $n \neq 3$?

A: Yes! $n=2$!