Integration by Parts: definite integrals

Recall: $\int F(x)g'(x)\,dx = F(x)g(x) - \int F'(x)g(x)\,dx$

for indefinite integrals.

For definite integrals you can

i) use the above to find the indefinite integral $F(x)$ and then evaluate $F(b) - F(a)$.

ii) Use the following:

~~$\int_a^b [F'(x)g(x) + F(x)g'(x)]\,dx$~~

$$\int_a^b F(x)g'(x)\,dx = \underbrace{\Big|_a^b F(x)g(x) - \int_a^b F'(x)g(x)\,dx}_{F(x)}$$

Example:

$$\int_0^{10} (1+0.4t)e^{-0.05t}\,dt$$

Choose: $F(t) = 1+0.4t$ and $g'(t) = e^{-0.05t}$

$F'(t) = 0.4$ $\quad g(t) = -\frac{1}{0.05}e^{-0.05t} = -20e^{-0.05t}$

$$\int_0^{10} (1+0.4t)e^{-0.05t}\,dt = \Big|_0^{10} -(1+0.4t)20e^{-0.05t} - \int_0^{10} 0.4(-20)e^{-0.05t}\,dt$$

$$= -(5)\cdot 20e^{-0.5} + 20 + 8\int_0^{10} e^{-0.05t}\,dt$$

$$= -100e^{-0.5} + 20 + 8\left(e^{-0.5} - 1\right)\frac{-1}{0.05}$$

$$= -100e^{-0.5} + 20 - 160\left(e^{-0.5} - 1\right) = 180 - 260e^{-0.5} \approx 22,3.$$

Integration by substitution: definite integrals

You need to change the limits of integration as well as the integral!

Example: $\int_{2}^{3} e^{x^2} 2x\,dx$ ($x^2 \to u$, $2x\,dx \to du$)

$u = x^2 \quad du = 2x\,dx$

$u(3) = 3^2 = 9$
$u(2) = 2^2 = 4$

$\Rightarrow \int_{4}^{9} e^u\,du$

Two options!

i) Calculate the indefinite integral first and then substitute in $u = g(x)$.

(ii) Do the full substitution including limits!)