

ECON3120/4120 Mathematics 2

Tuesday 2 June 2009, 14:30–17:30.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given: A (best), B, C, D, F, with D as the weakest passing grade.

Problem 1

The function f is defined over the entire xy -plane by

$$f(x, y) = e^{3x} + 3ye^x - y^3.$$

- Find the first and second order partial derivatives of f .
- Find the stationary points of f , if any, and determine whether they are local maximum points, local minimum points or saddle points.
- The level curve $f(x, y) = 3$ passes through the point $(x, y) = (0, -2)$. Find an equation for the tangent to the level curve at this point.

Problem 2

Let $f(x) = x^2e^x$ for all x .

- Over which one of the intervals $I_1 = (-\infty, -2)$, $I_2 = (-\infty, 0)$, and $I_3 = (-2, \infty)$ does f have an inverse function?
- Let g be the inverse function of f and let x_0 be a point where $f'(x_0) \neq 0$. Find an expression for $g'(f(x_0))$.

(Cont.)

Problem 3

- (a) Use Gaussian elimination to find a necessary and sufficient condition for the linear equation system

$$\begin{aligned}x + y - 3z &= a \\x - 3y + 4z &= b \\3x - y - 2z &= c\end{aligned}$$

to have at least one solution.

- (b) Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & 5 \\ r & 3 & -1 \\ 1 & s & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 2 & t & -19 \\ 1 & -4 & u \\ -1 & 5 & 11 \end{pmatrix}.$$

Calculate the matrix product \mathbf{AB} . If $\mathbf{B} = \mathbf{A}^{-1}$, what are the values of r , s , t , and u ?

Problem 4

- (a) Find the integral $\int \frac{t+1}{t(1+te^t)} dt$.

(*Hint*: Try the substitution $u = 1 + te^t$.)

- (b) Find the general solution of the differential equation

$$t(1 + te^t)\dot{x} = x^2(1 + t). \quad (*)$$

- (c) The differential equation (*) has a solution curve that passes through $(1, 1)$. Find an equation for the tangent to this solution curve at that point.