Compulsory term paper 1 in ECON3120/4120 Mathematics 2

Handed out: Monday 15 March 2004

To be delivered by: Thursday 25 March 2004

Place of delivery: Next to SV-info-center, ground floor.

Further instructions:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must a preprinted front page, which you will find at *http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc*
- It is important that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline **will not be read or marked.**$^{*)}$
- All term papers must be delivered to the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on the 12$^{\text{th}}$ floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

$^{*)}$ If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Problem 1

Given the function f defined by

$$f(x) = \frac{9e^x}{e^x + 1} - 2x - \frac{9}{2} \qquad \text{for all } x.$$

(a) Find the points where $f'(x) = 0$.

(b) Find the local maximum and minimum points of the function.

(c) Does the function have any global extreme points?

(d) Show that the function has exactly 3 zeros, and that they all lie in the interval $[-3, 3]$.

(Forts.)

Problem 2

Consider the following equation system:

$$\begin{aligned} x + ky + 3z &= 2 \\ kx + 9y + kz &= 3 \\ 7x + ky + 5z &= -6 \end{aligned}$$

(a) For what values of k does the system have one solution, several solutions, no solutions, respectively?

(b) Find all solutions of the system when it has solutions.

Problem 3

(a) Find the integrals (i) $\displaystyle\int \frac{x^3+2}{x}\,dx$ (ii) $\displaystyle\int_0^1 \sqrt{1-x}\,dx$.

(b) Give a geometric interpretation of $\displaystyle\int_0^a \sqrt{r^2-x^2}\,dx$ when $a \in (0, r]$. Then find the value of the integral for $a = r$.

(c) Find the indefinite integral $\displaystyle\int \frac{\ln x}{\sqrt{x}}\,dx$.

Problem 4

Find the limits

$$\text{(i)} \quad \lim_{x\to 0} \frac{\ln(e^x + x)}{x} \qquad \text{and} \qquad \text{(ii)} \quad \lim_{x\to\infty} \frac{\ln(e^x + x)}{x}$$

if they exist.

Problem 5

Let

$$F(T) = \int_0^T f(t)e^{-rt}\,dt + S(T)e^{-rT} - A,$$

where r and A are positive constants and f and S are differentiable functions. Find an expression for $F'(T)$, and show that if $F'(T)$ equals 0, then $f(T) = rS(T) - S'(T)$.