

## Compulsory term paper 2 in ECON3120/4120 Mathematics 2

Handed out: Thursday 15 April 2004

To be delivered by: Thursday 29 April 2004

Place of delivery: Next to SV-info-center, ground floor.

Further instructions:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, which you will find at [http://www.oekonomi.uio.no/info/EMNER/Forside\\_obl\\_eng.doc](http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc)
- It is important that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline **will not be read or marked.**<sup>\*)</sup>
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on the 12<sup>th</sup> floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

<sup>\*)</sup> If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

### Problem 1

Consider the function  $f$  defined by

$$f(x) = (ax + 1)e^{-bx} \quad (a, b \text{ constants, } a > b > 0)$$

- Compute  $f'(x)$  and  $f''(x)$ .
- Examine where  $f$  is increasing and where  $f$  is decreasing. Show that  $f$  has a maximum point  $x^*$ , and find this point. Show that  $x^{**} = x^* + 1/b$  is an inflection point for  $f$ .
- Examine the limits of  $f(x)$  when  $x$  approaches  $\infty$  and when  $x$  approaches  $-\infty$ .
- Compute  $\int_0^{\infty} f(x) dx$ .

(Cont.)

**Problem 2**

The equation

$$ze^z - xy = 0$$

defines  $z$  as a function of  $x$  and  $y$  in a neighbourhood of the point  $(x, y, z) = (1, e, 1)$ . Find  $z'_1(1, e)$ ,  $z'_2(1, e)$ , and  $z''_{12}(1, e)$ .

**Problem 3**

A certain model of wage formation contains the integral  $\int \frac{w}{(1-w)^3} dw$ . Compute this integral.

**Problem 4**

Consider the matrix  $\mathbf{A} = \begin{pmatrix} 11 & -6 \\ 18 & -10 \end{pmatrix}$ .

(a) Compute  $|\mathbf{A}|$ . Show that for a suitable number  $c$  we have  $\mathbf{A}^2 + c\mathbf{A} = 2\mathbf{I}_2$ .

(b) Show that there is no  $2 \times 2$  matrix  $\mathbf{B}$  such that  $\mathbf{B}^2 = \mathbf{A}$ .

**Problem 5**

The equation system

$$y = ax + bz$$

$$x = ce^y$$

defines  $y$  and  $z$  as functions of  $x$ . Find the elasticities of  $y$  and  $z$  with respect to  $x$ . Show that they can be expressed in terms of  $x$ .