ECON3120/4120 – Mathematics 2: **Problems for seminar 6, Mar. 27 & 31, Apr. 2 2008**

**1** (Exam problem 30/5-05)

(a) Calculate the determinant of $\mathbf{A}_t = \begin{pmatrix} 0 & t & 1 \\ 4 & -2 & 8 \\ 1 & 1 & 1 \end{pmatrix}$

(b) Find $x$, $y$ and $z$ such that

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} - \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix}$$

**2** Exam problem 142.

**3** Find the general solution of the differential equation

$$\dot{x} + \frac{2}{t}x = e^t$$

Find, in particular, the integral curve passing through $(t, x) = (1, 1)$.

**4** Given the matrix

$$\mathbf{A}_t = \begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$$

(a) Calculate $|\mathbf{A}_t|$ and show that $\mathbf{A}_t^{-1}$ exists for all $t$.

(b) Show that for a certain value of $t$ we have $\mathbf{A}_t^3 = \mathbf{I}_3$, where $\mathbf{I}_3$ is the identity matrix of order 3.

(c) Find the inverse of $\mathbf{A}_1$.

(d) Suppose that $\mathbf{A}$ and $\mathbf{B}$ are invertible $n \times n$-matrices. Show that if $\mathbf{A}'\mathbf{A} = \mathbf{I}_n$, then $(\mathbf{A}'\mathbf{B}\mathbf{A})^{-1} = \mathbf{A}'\mathbf{B}^{-1}\mathbf{A}$.