University of Oslo
Department of Economics

Obligatorisk oppgavesett nr. 1 i ECON3120/4120 Matematikk 2 / Compulsory term paper no. 1 in ECON3120/4120 Mathematics 2

Oppgavesettet foreligger kun på engelsk, men besvarelsene kan skrives på engelsk eller norsk.
This problem set is only available in English, but your term papers may be written in English or Norwegian.

Handed out: Friday, February 15th, 2007

To be handed in: Thursday March 06th, 2008 at 1400 hours. (Or earlier.)

Hand in at the department office, 12th floor

Other information:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, available in English resp. Norwegian at `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc` or `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_nor.doc`
- It is important that the term paper is submitted by the deadline (see above). Term papers submitted after the deadline **will not be read or marked.***)
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

There are four problems, one page

Problem 1, weight 30%. Consider the function $f(x) = \ln\sqrt{1+x^2} + e^x + e^{-x} - K$ defined for $x \geq 0$. Observe that $f'(0) = 0$.

(a) Show that f is convex. You might want to use the relation $\frac{1-x^2}{1+x^2} = \frac{2}{1+x^2} - 1$.

(b) Find, for each value of K, the number of zeroes. (Do not find the zeroes themselves.)

(c) Show that f has an inverse g and that $\lim\limits_{y \to (2-K)^+} g'(y) = +\infty$.

Problem 2, weight 30%.

(a) Find $\lim\limits_{x \to +\infty} (x^p - x^q)$ when $p > q > 0$.

(b) Calculate $\displaystyle\int \frac{x^6 - 6x^5 + 11x^4}{(x-1)(x-2)(x-3)}\,dx$. (Hint: Expand the denominator in order to divide, use the product from there on.)

(c) Calculate $\displaystyle\int_{e^e}^{1+e^e} \frac{1}{x} \cdot \frac{1}{\ln x} \cdot \frac{1}{\ln\ln x}\,dx$. (Hints: Substitute. The integral is improper.)

(d) Show that $\displaystyle\int_{-e}^{e} x^{314159265}\,|x|^{|x|^{|x|}} \ln(1 + x^{2008})\,dx = 0$.

(e) Consider the function $f(y)$ defined for all $y \geq 0$ by $f(y) = \int_0^\infty t^y e^{-t}\,dt$ (the integral is with respect to t as if y is constant.)
Show that $f(y+1) = (y+1)f(y)$. Hints:

- What does $\int_0^\infty$ mean?
- Use integration by parts
- You might want to look up MA1 p. 224 or EMEA p. 265.

Problem 3, weight 25%. Let $K > 0$ be a constant. Consider the differential equation

$$\dot{x} = (t - K)\frac{x}{\ln x} \qquad (\text{for } t > 0,\ x > 1)$$

(a) Find a t which is a stationary point for every solution $x(t)$.

(b) Find the solution which is such that $x(K) = e$.

Problem 4, weight 15%. Indicate true or false; justify briefly, but only briefly (that is: less than you would do for an exam).

(a) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$ matrices, then $(\mathbf{A} + \mathbf{B})' = \mathbf{A}' + \mathbf{B}'$.

(b) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$ matrices, then $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2$. *(Hint: Calculate the difference between the LHS and the RHS. Is it always zero?)*

(c) If the price vector $\mathbf{p}$, the initial endowment $\mathbf{a}$, and the post-trade endowment $\mathbf{x}$ are all n-vectors, with all prices $p_i > 0$ and $\mathbf{a} \neq \mathbf{0}$, then the budget constraint equation $\mathbf{p} \cdot (\mathbf{x} - \mathbf{a}) = 0$ in the unknown $\mathbf{x}$, has $n - 1$ degrees of freedom.