University of Oslo
Department of Economics

Obligatorisk oppgavesett nr. 2 i ECON3120/4120 Matematikk 2 / Compulsory term paper no. 2 in ECON3120/4120 Mathematics 2

Oppgavesettet foreligger kun på engelsk, men besvarelsene kan skrives på engelsk eller norsk.
This problem set is only available in English, but your term papers may be written in English or Norwegian.

Handed out: Friday, April 11th, 2008

To be handed in: Monday April 28th, 2008 at 1400 hours. (Or earlier.)

Hand in at the department office, 12th floor

Other information:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, available in English resp. Norwegian at `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc` or `http://www.oekonomi.uio.no/info/EMNER/Forside_obl_nor.doc`
- It is important that the term paper is submitted by the deadline (see above). Term papers submitted after the deadline **will not be read or marked.***)
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

There are four problems which will be given approximately equal weight

Problem 1 Consider for each u and each k the following linear equation system in the unknowns x, y and z:

$$\mathbf{A}_u \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ k \\ ku \end{pmatrix} \qquad \text{where} \qquad \mathbf{A}_u = \begin{pmatrix} 1 & 2u-1 & 1-u \\ u-1 & 1 & 3u-1 \\ 0 & u & 2u \end{pmatrix}$$

Determine, for all values of u and k whether the equation system (i) has a solution, and (ii) if there is a solution, whether it is unique, and (iii) if there is more than one solution, the number of degrees of freedom.

Problem 2 In a model for optimal thinning of a growing forest, one encounters the problem of maximizing the twice continuously differentiable function

$$V(t,x) = g(t)h(x)e^{-rt} - x,$$

where $g(t)$ and $h(x)$ are strictly positive functions defined for $t > 0$, $x > 0$. r is stricly positive.

(a) What are the first-order conditions for $V(t,x)$ to have a maximum at (t^*, x^*)?

(b) It will follow that $V''_{tx}(t^*, x^*) = 0$ (but you are not required to show it).
Show that if $h''(x^*) < 0$ then the point (t^*, x^*) satisfies the local second-order conditions for a maximum point if

$$g''(t^*) < r^2 g(t^*).$$

(Hint: Use the first-order condition for V'_t.)

(c) Find t^* and x^* when $g(t) = e^{\sqrt{t}}$ and $h(x) = \ln(x+1)$, and check the local second-order conditions.

Problem 3 The equation system

$$x \cdot y \cdot z \cdot ue^{-u} \cdot ve^{v} = 2$$
$$1x + 2y + 3z + 4u + 5v = 6$$

defines u and v implicitly as continuously differentiable functions of (x,y,z) around the point $P:\ (x,y,z,u,v) = (2,-1,-1,1,1)$. (You are not supposed to prove this.)

(a) Differentiate the system (i.e. find the differentials).

(b) Find a general expression for v'_x.

(c) Find an approximation for u in the point where $x = 2$, $y = -1$ and $z = -1.1$.

Problem 4 Consider the problem

$$\max_{(x,y)} \; 4e^x + \tfrac{1}{2}Ax^2y^2 + e^{3y} \qquad \text{subject to} \qquad \begin{cases} x^2 + By^2 & \leq C \\ x & \geq 0 \\ y & \geq 0 \end{cases} \tag{P}$$

where A, B and C are strictly positive constants.

(a) State the Kuhn-Tucker conditions associated with the problem.

(b) Show that the Kuhn-Tucker conditions imply $x^2 + By^2 = C$ and $xy \neq 0$.

Note: Do *not* try to solve the problem (P)!