University of Oslo
Department of Economics

ECON3120/4120 Mathematics 2, spring 2009

Problems for Seminar 1, 26–30 January 2009

1 Consider the function f defined by

$$f(x) = \frac{3-x}{3x-3}$$

(a) Where is $f(x)$ defined? Calculate $f(x)$ for $x=-3$, $x=-1/2$, $x=1/4$, $x=3/2$, $x=3$, and $x=9$.

(b) Where is $f(x) \le 0$? Where is $f(x) \le 1$?

(c) Draw the graph of f and see if your answers to (b) are confirmed.

(d) Define $g(x) = \ln[f(x)]$. Where is $g(x)$ defined? Where is $g(x) > 0$?

2 Use l'Hôpital's rule (or other methods) to find the limits:

(a) $\displaystyle\lim_{x\to 3} \frac{3x^2-27}{x-3}$ (b) $\displaystyle\lim_{x\to 0} \frac{e^{-3x}-e^{-2x}+x}{x^2}$ (c) $\displaystyle\lim_{x\to\infty} \left(\sqrt{x^2+\tfrac{1}{2}x}-x\right)$

3 (a) The equation $e^L + KL = Ke^K$ defines L as a differentiable function of K. Find an expression for dL/dK.

(b) If $z = F(u,v,w)$ and $u = f(x,y)$, $v = e^{-x}$, and $w = \ln y$, find an expression for $\partial z/\partial x$ and $\partial z/\partial y$.

4 **EMEA:** 7.10.1 = **MA I:** 6.6.1

5 **Exam problem 28:**

The function g is given by $g(x) = 2x - ae^{-x}(1+x^2)$, where a is a positive constant.

(a) Determine where the function g is convex.

(b) Find $\lim_{x\to\infty} g(x)$. Show that $g(x)=0$ has exactly one solution, x_0, and that $x_0 > 0$.

(c) Show that $x_0 < a/2$. (*Hint:* Show that $g'(x) > 2$ for $x \ne 1$.)

(d) Define the function f by $f(x) = ae^{-x} + \ln(1+x^2)$. Show that the point x_0 that you found in (b) is a global minimum point of f.

(e) The point x_0 defined by the equation $g(x_0) = 0$ depends on a. Find an expression for dx_0/da.

(f) Compute $\displaystyle\lim_{a\to 0^+} (x_0/a)$.

(*Note:* The Norwegian version of the exam problem collection is for sale in "Kopiutsalget" in the Akademika bookstore. If you cannot read Norwegian, please contact Arne Strøm for an English version.)