

ECON3120/4120 Mathematics 2, spring 2009

Problems for Seminar 3, 9–13 February

1 EMEA: 7.5.5 = **MA I:** 7.4.5

2 (Problem 63(a) in the compendium of exam problems.) The equation

$$3xe^{xy^2} - 2y = 3x^2 + y^2$$

defines y as a differentiable function of x around the point $(x^*, y^*) = (1, 0)$. Find the slope of the graph at this point by implicit differentiation. What is the linear approximation to y around $x^* = 1$?

3 Assume that the marginal cost function of a firm is

$$C'(x) = x^2 + x - 10$$

and that the fixed costs are 50. Find the cost function.

4 Evaluate $\int_0^2 2x^2(2-x)^2 dx$. Give a rough check of the answer by sketching the graph of $f(x) = 2x^2(2-x)^2$ over $[0, 2]$.

5 EMEA: 9.5.1(d) = **MA I:** 10.6.1(d)

6 EMEA: 9.6.2(c) = **MA I:** 10.7.2(c)

7 Consider the function f defined by $f(x) = x(\ln x)^2$ for all $x > 0$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Decide where f is increasing and where f is decreasing. Does f have global extreme points?

(c) Find $\int x(\ln x)^2 dx$.

8 Show that $\int \sqrt{x^2 + 3} dx = \frac{1}{2}x\sqrt{x^2 + 3} + \frac{3}{2}\ln(x + \sqrt{x^2 + 3}) + C$.

(*Hint:* Think carefully about what you are asked to do.)