University of Oslo / Department of Economics **English**

ECON3120/ECON4120 Mathematics 2

Friday December 7 2007, 09:00–12:00

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Give reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1 Consider for each u and each k the following linear equation system in the unknowns x, y and z:

$$\mathbf{A}_u \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ k \\ ku \end{pmatrix} \qquad \text{where} \qquad \mathbf{A}_u = \begin{pmatrix} 1 & 2u-1 & 1-u \\ u-1 & 1 & 3u-1 \\ 0 & u & 2u \end{pmatrix}$$

Determine, for all values of u and k whether the equation system (i) has a solution, and (ii) if there is a solution, whether it is unique, and (iii) if there is more than one solution, the number of degrees of freedom.

Problem 2 In a model for optimal thinning of a growing forest, one encounters the problem of maximizing the twice continuously differentiable function

$$V(t,x) = g(t)h(x)e^{-rt} - x,$$

where $g(t)$ and $h(x)$ are strictly positive functions defined for $t > 0$, $x > 0$. r is stricly positive.

(a) What are the first-order conditions for $V(t,x)$ to have a maximum at (t^*, x^*)?

(b) It will follow that $V''_{tx}(t^*, x^*) = 0$ (but you are not required to show it).
Show that if $h''(x^*) < 0$ then the point (t^*, x^*) satisfies the local second-order conditions for a maximum point if

$$g''(t^*) < r^2 g(t^*).$$

(Hint: Use the first-order condition for V'_t.)

(c) Find t^* and x^* when $g(t) = e^{\sqrt{t}}$ and $h(x) = \ln(x+1)$, and check the local second-order conditions.

Problem 3 Let $K > 0$ be a constant. Consider the differential equation

$$\dot{x} = (t-K)\frac{x}{\ln x} \qquad (\text{for } t > 0,\ x > 1)$$

(a) Find a t which is a stationary point for every solution $x(t)$.

(b) Find the solution which is such that $x(K) = e$.

Problem 4 Consider the problem

$$\max_{(x,y)} \; 4e^{x} + \tfrac{1}{2}Ax^2y^2 + e^{3y} \qquad \text{subject to} \qquad \begin{cases} x^2 + By^2 & \leq C \\ x & \geq 0 \\ y & \geq 0 \end{cases} \tag{P}$$

where A, B and C are strictly positive constants.

(a) State the Kuhn-Tucker conditions associated with the problem.

(b) Show that the Kuhn-Tucker conditions imply $x^2 + By^2 = C$ and $xy \neq 0$.

Note: Do *not* try to solve the problem (P)!