University of Oslo
Department of Economics

English

ECON3120/4120 Mathematics 2

Monday 4 June 2007, 09.00–12.00.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given: A (best), B, C, D, F, with D as the weakest passing grade.

Oppgave 1

For every real number a we define the matrix $\mathbf{A}_a$ by

$$\mathbf{A}_a = \begin{pmatrix} 3 & 2 & -4 \\ 1 & 1 & 2a-3 \\ 2 & a & 2 \end{pmatrix}.$$

(a) Find the determinant $|\mathbf{A}_a|$.

(b) For what values of a does the equation system

$$\begin{aligned} 3x + 2y - 4z &= 2 \\ x + y + (2a-3)z &= 3 \\ 2x + ay + 2z &= 6 \end{aligned}$$

have (i) exactly one solution, (ii) several solutions, (iii) no solutions?

Oppgave 2

Find the general solution of the differential equation

$$\dot{x} - x = \frac{e^t}{t}, \qquad t > 0.$$

Also find the particular solution that gives $x = e^{-1}$ for $t = 1$.

(Cont.)

Oppgave 3

The equation system

$$x + e^{v-u} - \ln y = 1$$
$$xy - u + 2v^2 = e$$

defines u and v as continuously differentiable functions of x and y around the point $(x, y, u, v) = (1, e, 0, 0)$. (You are not supposed to prove this.)

(a) Differentiate the system (i.e. calculate differentials).

(b) Find a general expression for v'_y.

Oppgave 4

In this problem A, a, and b are constants, with $A > 0$ and $a \neq 0$. The equation

$$u + \ln u = Ax + \frac{1}{2}y^2$$

defines u implicitly as a function $u = u(x, y)$ of x and y. (You are not supposed to prove this.)

(a) Find expressions for the partial derivatives $u'_1(x, y)$ and $u'_2(x, y)$.

(b) Use Lagrange's method to set up the necessary first-order conditions for a point (x, y) to solve the problem

(P) maximize/minimize $ax + by$ subject to $u(x, y) = K$,

where K is a constant. Show that there exists exactly one point (x^*, y^*) that satisfies these conditions.

(c) Show that the level curve $u(x, y) = K$ consists of all points (x, y) for which $y = \pm\sqrt{Q - 2Ax}$, where Q is a constant that depends on K.

(d) (Difficult.) Use the result from part (c) to decide for what values of a and b the point (x^*, y^*) that you found in part (b) is a maximum point in problem (P). (*Hint:* Draw the level curve $u(x, y) = K$ together with some level curves for $ax + by$.)