

ECON3120/4120 Mathematics 2

Friday, 9 December 2005, 09:00–12:00

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Give reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1

Consider the function f defined by

$$f(x) = ae^{2x} - be^x \quad \text{for all } x,$$

where a and b are constants with $0 < a < b$.

- Show that f has exactly one zero, x_0 , and exactly one stationary point, x_1 . Show also that $x_0 - x_1$ is independent of a and b .
- Where is the function f positive, where is it increasing, and where is it decreasing?
- Investigate $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow \infty} f(x)$.

Problem 2

Let f be a function of two variables given by

$$f(x, y) = (x^2 - y)^2 + x^3 - 3x^2 \quad \text{for all } x \text{ and } y.$$

- Calculate the first and second order partial derivatives of f .
- Find all the stationary points of f , and classify them by means of the second-derivative test.
- Does f have any global extreme points?

(Cont.)

Problem 3

- (a) Calculate the integral $\int_0^{\sqrt{15}} 3x\sqrt{1+x^2} dx$.
- (b) Find the general solution of the differential equation $e^{-t}\dot{x} = x^2 \ln(1 + e^t)$.

Problem 4

Consider the matrices

$$\mathbf{A}_a = \begin{pmatrix} 1 & 2 & 2 \\ 1 & a & -1 \\ 1 & 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{B}_b = \begin{pmatrix} b & -2 & 12 \\ 2 & 1 & -3 \\ 3 & 0 & -3 \end{pmatrix}.$$

- (a) Calculate the determinant $|\mathbf{A}_a|$. For what values of the parameter a does \mathbf{A}_a have an inverse?
- (b) Show that for suitable values of a and b the product matrix $\mathbf{A}_a\mathbf{B}_b$ is a diagonal matrix. Use your result to find the inverse matrix of $\begin{pmatrix} 1 & 2 & 2 \\ 1 & 5 & -1 \\ 1 & 2 & 1 \end{pmatrix}$.