University of Oslo
Department of Economics

English

ECON3120/4120 Mathematics 2

Monday 8 December 2008, 09:00–12:00.

There are 2 pages of problems to be solved.

All printed and written material may be used. Pocket calculators are allowed.

State reasons for all your answers.

Grades given: A (best), B, C, D, E, F, with E as the weakest passing grade.

Problem 1

Consider the matrix $\mathbf{A}_t = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & t & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$, where t is an arbitrary real number.

(a) Calculate the determinant $|\mathbf{A}_t|$. For what values of t will $\mathbf{A}_t$ have an inverse?

(b) Show that $\mathbf{A}_t + \mathbf{A}_s = 2\mathbf{A}_{(t+s)/2}$ and use this to calculate $|\mathbf{A}_t + \mathbf{A}_s|$.

(c) Does the equation system $\mathbf{A}_t\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ have a solution for every t?

(You need not find the solutions.)

Problem 2

The equation system

$$e^x y + u - v = 1$$
$$x - e^{u^2+v} + y = -e$$

defines u and v as continuously differentiable functions of x and y in an open set around the point P_0 with coordinates $(x, y, u, v) = (0, 0, 1, 0)$.

Find the values of u'_x, u'_y, v'_x, and v'_y at P_0.

(Cont.)

Problem 3

Let

$$f(x, y) = 10 \ln(x + 2y) + x - 22y - \tfrac{3}{2}x^2 + 6xy$$

for all x and y with $x + 2y > 0$.

Find the stationary points of f and determine for each of them if it is a local maximum point, a local minimum point or a saddle point.

Problem 4

(a) Find the general solution of the differential equation

$$\dot{x} - x = e^t - t \tag{$*$}$$

(b) Let K be the integral curve for $(*)$ that passes through $(t_0, x_0) = (1, 2)$. Find the tangent to K at the point $(1, 2)$, and show that this tangent has no other point in common with K.