University of Oslo
Department of Economics

Voluntary term paper in ECON3120/4120 Mathematics 2

Announced: Friday 6 March 2009.

To be handed in during the lecture on Thursday 26 March 2009.

If you are not able to come to that lecture, you may hand in your paper at the department reception on the 12th floor.

Further instructions:

- This term paper is **voluntary** and is meant to give you a little practice in writing answers to mathematics problems.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- The problems are given only in English, but you are free to write your answers in Norwegian, Danish, Swedish, or English.)
- Please write your answers as clearly as possible, giving reasons for your statements. They should be written in such a way that another student can understand them. And please write legibly!
- If you happen to recognize a problem as one you have seen before, don't look up the old answer, but do the problem as if you haven't seen it before.

Please hand in this sheet together with your answers. You will get it back.

Name (block letters, please!):

Problem 1

(a) Solve the equation $3^x 2^{3x} = 17$.

(b) Find all solutions (x, y) of the equation system

$$\begin{aligned} 4x^2y^2 - x^2y^4 &= 0 \\ x + xy &= 0 \end{aligned}$$

Problem 2

(a) Calculate the integral $\displaystyle\int_{-2/3}^{1/3} \frac{x\,dx}{\sqrt{2-3x}}$.

(b) Find $\lim_{x \to 0^+} f(x)$, where $f(x) = x^{e/(1+\ln x)}$. (*Hint:* Look at $\ln f(x)$.)

(Cont.)

Problem 3

(a) Find all solutions of the equation system

$$\begin{aligned} x_1 + \ \ x_2 \qquad\quad &- 2x_4 = 2 \\ 2x_2 - x_3 &- \ \ x_4 = 3 \\ x_1 + \ \ x_2 \qquad\quad &+ \ \ x_4 = 2 \end{aligned}$$

(b) Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 0 & 2 & -1 & -1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Calculate the matrix product $\mathbf{AA}'$ and show that

$$(\mathbf{AA}')^{-1} = t \begin{pmatrix} 17 & -12 & 4 \\ -12 & 18 & -6 \\ 4 & -6 & 20 \end{pmatrix}$$

for a suitable value of t. Find t. Both $\mathbf{AA}'$ and $(\mathbf{AA}')^{-1}$ are symmetric. Is that just a coincidence?

Problem 4

(a) Find the general solution of the differential equation

$$\dot{x} - 2x = k - 2t\,, \tag{$*$}$$

where k is a constant.

(b) Show that if $x = x(t)$ is a function such that

$$x(t) = 2\int_0^t x(s)\,ds + t + 1 - t^2 \tag{$**$}$$

for all t, then x is a solution of the differential equation $(*)$ in part (a) for a suitable value of k. Use this to find a function $x(t)$ that satisfies $(**)$.

Problem 5

Let f be defined by $f(x) = \dfrac{4a}{1+a^2}(1 - e^{-x}) - x$ for all x (a is a constant).

(a) Calculate $f'(x)$, $f''(x)$ and $\lim_{x\to\infty} f(x)$.

(b) For what values of a will the equation $f(x) = 0$ have a solution $x_0 > 0$?

(c) Find the value of x that maximizes $f(x)$ over $[0, \infty)$.

(d) Show that if f attains its maximum over $[0, \infty)$ at the point x_1, then $f(x_1) = e^{x_1} - 1 - x_1$.