Seminar 11, week 17

Approximation

Exercise 1

Calculate the Taylor formula up to polynomial power 2 and 3 of the following functions, and compare the approximate values with the function values at the point $x = 2$. Which is the best approximate of the two powers?

a) $f(x) = xe^x$

b) $f(x) = \frac{1}{1+x}$

c) $f(x) = \frac{1}{(1+x)^2}$

Homogeneous and homothetic functions

Exercise 2

Find the degree of homogeneity of the following funcions

a) $x^4 + x^2y^2$

b) Ax^ay^b

c) $\frac{\sqrt{x}+\sqrt{y}+\sqrt{z}}{x+y+z}$

Exercise 3

Study the homogeneity property of $f(x, y) = \frac{xy}{x^2+y^2}$, and examine the Euler's homogeneous function theorem.

Exercise 4

Decide the degree of homogeneity of

$$f(x, y) = a\ln(\frac{g(x, y)}{x})$$

when the degree of homogeneity of $g(x, y)$ is equal to 1.

Exercise 5

Is the function $F(x, y) = xy + 1$ homogeneous? Is it homothetic?

Kuhn-Tucker

Exercise 6

a) Find the best candidate for the problem of maximizing $f(x, y) = \ln(x+1) + \ln(y+1)$ subject to $x + 2y \leq 5/2$ and $x + y \leq 2$.

b) Verify that this candidate solves the problem (i.e. satisfies some sufficient condition).

Exercise 7

Autumn Exam 2006, Problem 2b.