Econ4130 14H

Exercises for no-seminar week 37

(solutions on the net at the end of the week)

Rice chapter 2: (on the Weibull distribution)

33

67b,c + extra for c.: Suppose you have drawn two (by calculator e.g.) observations, 0.08 and 0.63, from the uniform[0, 1] distribution. Transform these numbers to two observations drawn from a Weibull distribution with $\alpha = 2$ and $\beta = 3$.

Rice chapter 3:

1 + extra: Calculate the regression function, $E(Y \mid x)$ (i.e. set up a table of $E(Y \mid x)$ for x = 1,2,3,4).

10 + extra: Find the regression functions, $E(Y \mid x)$ and $E(X \mid y)$. Are they linear ? Are they homoscedastic or heteroscedastic? [**Hint:** Identify both of the conditional distributions as gamma-distributions.]

18 + extra: Find $E(Y \mid x)$ [**Hint:** For **18b,c,** read **general hints** in the appendix below.**]**

Appendix

General hints: **Integration over non rectangular areas (for exercise 3.18).**

In some of the exercises in this course you will have to calculate double integrals over areas that are not rectangles. It is not sure whether I will have time or not to talk about this in the lectures, so here is an example. There is nothing new involved - only that you need to be a bit careful with the integration limits.

We will look at an example from the lectures. Suppose $(X,Y) \sim f(x,y)$, where the pdf, *f* is

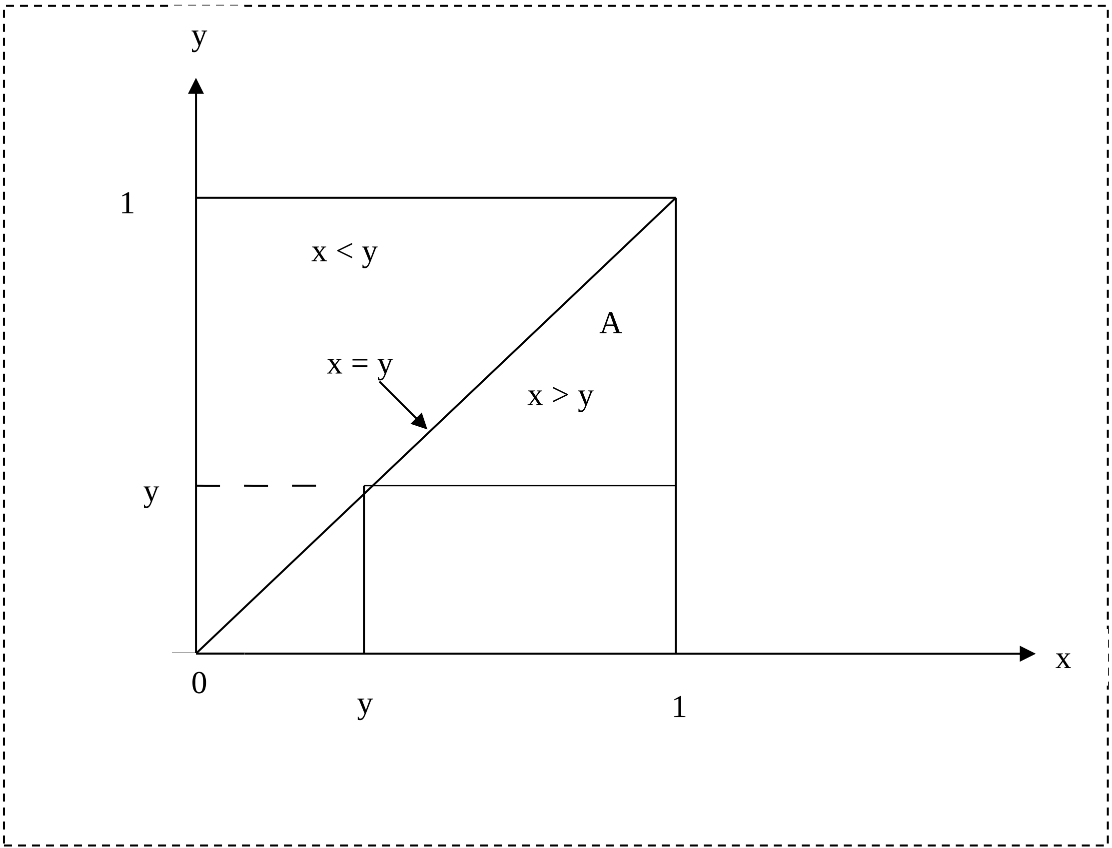
$$f(x,y) = \begin{cases} \frac{12}{7}(x^2 + xy) & \text{for } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

What is $P(X > Y)$?

(*Solution next page*)

Solution: Look at figure 1. What we ask for is the probability that an observation of (X,Y) will be a point falling in the lower triangle called *A* on the figure, consisting of all points (x,y) (within the square of possible observations) where $x>y$.

Figure 1



According to the theory this probability is the volume under the pdf over that area, i.e., the integral over *A* of *f*:

$$P(X>Y)=P((X,Y)\in A)=\iint_A f(x,y)dxdy=\int_0^1\left[\int_y^1 f(x,y)dx\right]dy$$

Explanation of the inner integral: In the inner integral you integrate with respect to *x* while keeping *y* fixed. Now proceed as follows: Fix first a *y* somewhere arbitrary between 0 and 1 on the y-axis (see the figure). Then find out which *x*'s (on the x-axis) are such that (x,y) belongs to *A* for that particular *y*. Looking at the figure we see that all *x* such that $y\le x\le 1$ satisfy this. Hence the inner integral must be over the interval $[y,1]$, i.e., $\int_y^1 f(x,y)dx$, giving

$$\int_y^1 f(x,y)dx = \frac{12}{7}\int_y^1 (x^2+xy)dx = \frac{12}{7}\Big|_y^1\left(\frac{1}{3}x^3 + y\frac{1}{2}x^2\right) = \frac{12}{7}\left[\frac{1}{3}+\frac{y}{2}-\frac{y^3}{3}-\frac{y^3}{2}\right] = \frac{12}{7}\left[\frac{1}{3}+\frac{y}{2}-\frac{5}{6}y^3\right] =$$

$$= \frac{2}{7}\left[2+3y-5y^3\right] \qquad \text{for any chosen } y \text{ in the interval } [0,1]$$

Having found the inner integral as a function of *y*, we can now integrate that function over all values of *y* where the pdf, *f*, can be > 0, i.e., over the interval $[0,1]$.
Hence

$$P(X>Y) = \iint_A f(x,y)dxdy = \int_0^1\left[\int_y^1 f(x,y)dx\right]dy = \frac{2}{7}\int_0^1\left(2+3y-5y^3\right)dy =$$

$$= \frac{2}{7}\Big|_0^1\left[2y+\frac{3}{2}y^2-\frac{5}{4}y^4\right] = \frac{2}{7}\left[2+\frac{3}{2}-\frac{5}{4}\right] = \frac{2}{7}\cdot\frac{8+6-5}{4} = \frac{9}{14}$$

You could also, of course, have integrated the other way, with respect to *y* first (fixing *x* on the x-axis), and then with respect to *x*. Do that yourself for practice, and check that you get the same answer.

You may also practice on Rice exercise 3:8a. for which I give the answers (and hope that I calculated correctly (!)): $P(X>Y)=1/2$, $P(X+Y\le 1)=3/14$, $P(X\le \frac{1}{2})=2/7$.