

HG  
Nov. 16

## ECON 4130 16H

### Exercises for seminar week 46

- Rice chapter 9:** No. 12, 33 (**Hint:** note that there are 0 parameters under  $H_0$  here, so the DF for the Chi-square test must be equal to the number of free parameters in the full model.)  
 No. 40 (Remember that  $Z \sim N(0,1) \Rightarrow Z^2 \sim \chi_1^2$  - distributed. )  
 (See, e.g., Rice, example C, sec 2.3, p.61)  
 No. 41

#### Extra exercise

An econometric model contains a response,  $Y$ , and 6 (exogenous) explanatory variables,  $X, Z_1, Z_2, U_1, U_2, U_3$ . The data are observations of  $n = 22$  iid corresponding random vectors,  $(Y_i, X_i, Z_{i1}, Z_{i2}, U_{i1}, U_{i2}, U_{i3})$ , and the (full) regression model is (using observed values of the explanatory variables as fixed)

$$(1) \quad Y_i = \alpha + \beta x_i + \delta_1 z_{i1} + \delta_2 z_{i2} + \gamma_1 u_{i1} + \gamma_2 u_{i2} + \gamma_3 u_{i3} + e_i \quad \text{for } i = 1, 2, \dots, 22$$

Where,  $e_1, e_2, \dots, e_n$  are iid and normal,  $e_i \sim N(0, \sigma^2)$ .

- A. Estimating (1) by OLS gives the following table of sums of squares (using Stata terminology)

**Table 1 (for full model)**

Source	SS	df
Model	7817	?
Residual	3743	?
Total	11560	?

Fill in the degrees of freedom (df's) in the table. Estimate the error term variance,  $\sigma^2$ , using an unbiased estimator.

- B. A submodel of interest is assuming both  $\delta_1 = \delta_2$  and  $\gamma_1 = \gamma_2 = \gamma_3$ . We want to check if there is evidence in the data against this submodel using an appropriate F-test. We then need to re-estimate the model assuming the submodel (that we call

the “reduced model”) to be true. Using OLS for the reduced model implies that we must regress the response  $Y$  on a modified set of explanatory variables.

Write up the corresponding (to (1)) regression model in the reduced case.

**[Hint:** Introduce two new parameters,  $\delta$  for the common value of  $\delta_1, \delta_2$ , and  $\gamma$  for the common value of  $\gamma_1, \gamma_2, \gamma_3$ , and substitute in (1). Define new regressor (i.e., explanatory) variables whenever necessary.]

- C. Estimating the reduced model by OLS gives the following table of sums of squares (using Stata terminology)

**Table 2 (for the reduced model)**

Source	SS	df
Model	5332	?
Residual	6228	?
Total	11560	?

Use this information to perform an F-test for testing the sub-model against the more general model in (1).

Calculate the P-value, either approximately using the quantile table 5 in the back of Rice’s book, or exactly using (e.g.) the “F.dist” function in Excel, or the  $F(df1, df2, f)$  – function (or  $Ftail(df1, df2, f)$ -function) in STATA.