Econ 4130
HG Nov. 2017

Correction note to the lecture note on F-testing

The section in the lecture note on page 12, starting with the heading
Example of testing structural break described in the introduction.
and until Section 5 on page 13, should be replaced by the following.

(The full model is correct, but the reduced model was wrong in the lecture note. The corrected F-test result, however, is quite similar to the one in the lecture note, and the conclusion the same.)

Full model

$Y_i = \beta_0 + \beta_1 x_i + \beta_2 d_i + \beta_3 d_i x_i + e_i$ where $e_1, e_2, \ldots, e_n \sim iid$ with $e_i \sim N(0, \sigma^2)$

Reduced model

$Y_i = \beta_0 + \beta_1 x_i + e_i$ where $e_1, e_2, \ldots, e_n \sim iid$ with $e_i \sim N(0, \sigma^2)$

The reduced model corresponds to

$H_0 : \beta_2 = \beta_3 = 0$ versus H_1 : At least one of β_2, β_3 different from 0.

The Stata data base contains the 4 regressor variables, $Y, x, d,$ and $xd = x \cdot d$.

Stata output full model (OLS)

Stata command: `regr Y x d xd`

```
      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(3, 16)        =     68.92
       Model |  5784808.74         3  1928269.58   Prob > F        =    0.0000
    Residual |  447637.457        16   27977.341   R-squared       =    0.9282
-------------+----------------------------------   Adj R-squared   =    0.9147
       Total |   6232446.2        19  328023.484   Root MSE        =    167.26

------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .2742643   .0459396     5.97   0.000     .1768768    .3716518
           d |   1639.755   283.2312     5.79   0.000     1039.331    2240.178
          xd |  -.2745789   .0572058    -4.80   0.000    -.3958499    -.153308
       _cons |   86.25502   105.3841     0.82   0.425    -137.1493    309.6594
```

Reduced model (H_0)

$Y_i = \beta_0 + \beta_1 x_i + e_i$ where $e_1, e_2, \ldots, e_n \sim iid$ with $e_i \sim N(0, \sigma^2)$

$\Leftrightarrow \quad H_0 : \beta_2 = \beta_3 = 0$

Stata command: `regr Y x`

Stata output reduced model (OLS)

```
      Source |       SS           df       MS      Number of obs   =        20
-------------+----------------------------------   F(1, 18)        =     62.89
       Model |   4845492.5         1   4845492.5   Prob > F        =    0.0000
    Residual |   1386953.7        18  77052.9834   R-squared       =    0.7775
-------------+----------------------------------   Adj R-squared   =    0.7651
       Total |   6232446.2        19  328023.484   Root MSE        =    277.58

------------------------------------------------------------------------------
           Y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .1748752   .0220523     7.93   0.000      .128545    .2212053
       _cons |   331.2991   102.3147     3.24   0.005      116.344    546.2543
------------------------------------------------------------------------------
```

The relevant quantities are

$SS_{full} = \quad 447\,637.457 \qquad df_{full} = 16$

$SS_{red} = \quad 1\,386\,953.7 \qquad df_{red} = 18$

No. of restrictions under H_0: $s = df_{red} - df_{full} = 2$

$$F = \frac{(SS_{red} - SS_{full}) / s}{SS_{full} / df_{full}} = \frac{(1\,386\,953.7 - 447\,637.457) / 2}{447\,637.457 / 16} = 16.787...$$

$F \sim F(2,16)$ under H_0.

P-value (using F.Dist in Excel): $P_{H_0}(F > F_{obs}) = P_{H_0}(F > 16.787) = 0.0001177$, i.e., 0.000, so the evidence for a structural break as defined at 5000 is strong, i.e., the reduced model is rejected.